ASPECTS OF

QUINTESSENCE MATTER -

THE DRIVER OF THE LATE TIME

ACCELERATION OF THE UNIVERSE

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SUDIPTA DAS

DEPARTMENT OF PHYSICS

JADAVPUR UNIVERSITY

KOLKATA, INDIA

CERTIFICATE FROM THE SUPERVISOR

This is to certify that the thesis entitled **"Aspects of Quintessence Matter - The Driver of the Late Time Acceleration of the Universe"**, submitted by **Sudipta Das** who got her name registered on **23rd April, 2004** for the award of **Ph.D (Sciences)** degree of **Jadavpur University**, is absolutely based upon her own work under the supervision of **Dr. Narayan Banerjee** and that neither this thesis nor any part of it has been submitted for any degree or any other academic award anywhere before.

(Dr. Narayan Banerjee)

Reader, Department of Physics

Jadavpur University, Kolkata - 700 032

India

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Department of Physics,

Jadavpur University,

Kolkata 700032,

India.

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Sudipta Das

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Preface

The last decade witnessed a radical change in cosmology - the science of the universe. Cosmology now becomes an observation dependent science, like other branches of physics. This is brought about by the high precision observation techniques developed over the last few years. The great surprise that results from the high precision data is the inference that the universe is undergoing an accelerated expansion at present defying all intuitions. The search for the matter responsible for this unexpected behaviour of the universe provides one of the greatest excitements in contemporary theoretical physics.

The present thesis is a collection of five papers based on my research on the problem of 'dark energy', the driver of alleged present acceleration of the universe. All the five papers are published in international journals.

The thesis is divided into five chapters. The first chapter is an introduction where the problem is defined and a survey of the work already in the literature has been made. A brief outline of the present work is also included in the introduction.

The next four chapters include the actual work done. The reprints of the published papers are presented as the chapters or sections thereof. Only the last chapter contains a slightly improved version of the published paper.

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Sudipta Das

Chapter 1

Introduction

The universe consists of everything that we can see through our best gadgets, have seen in the past and expect to see in any intelligible future. Cosmology deals with the physics of this most exhaustive collection of objects as a whole. Cosmology is concerned with the formation, the evolution, the future of the universe made of billions of galaxies spread over billions of light years.

Cosmology really came of age as a science after the advent of general relativity in 1915 which made possible a systematic theoretical modelling of the universe and Hubble's observation in 1929 that the universe is in fact evolving and hence interesting as a physical science. Like all other branches of physics, cosmology is also an observational science, but until very recently, the data available had been limited and that too had been plagued by the lack of precision. The dramatic change in the scenario started with the COSMIC BACKGROUND EXPLORER (COBE) [1, 2, 3, 4] and over the past decade there had been an explosion of really high precision data, such as those from WILKINSON MICROWAVE ANISOTROPY PROBE (WMAP) [5, 6, 7, 8, 9]. One inherent problem in cosmology, that one cannot repeat experiments, will remain for ever, one cannot ask the universe to evolve afresh with various initial conditions. But the data available on the existing system holds the key to dictate the direction of research in this branch.

Cosmology as a subject is as vast as the universe, and application of every branch of physics is indeed warranted. The recent advances in observational cosmology has led to many exciting discoveries and possibilities regarding the evolution of the universe, but arguably the most exciting and puzzling amongst them is that the universe now is expanding with an acceleration defying the properties of the known matter content of the universe. For a systematic and lucid review of the observational results and their interpretations, we refer to

the work by L. Perivolaropoulos [10]. The present thesis endeavours to look at this problem from a very narrow angle.

As already mentioned, this acceleration is counter intuitive as gravity, which is the deciding interaction in the governance of the dynamics of the universe as a whole, is always attractive. Gravity is by far the weakest amongst the four basic interactions of nature, but the strong and weak interactions are short range and have hardly anything to do with the dynamics of the universe at a large scale, while electromagnetic interaction has almost no impact in view of the charge neutrality of the universe. So the galaxies should attract each other, and even if the universe expands as suggested by the observations as well as the standard models of cosmology, the rate of expansion should be decreasing.

1.1 Standard Cosmological Model :

For an observationally realistic and logically viable model of the universe, one makes the following assumptions:

(1) General Relativity (GR) correctly describes gravity.

(2) Cosmological principle is valid, i.e, universe on a large scale ($> 10^6$ light years) is spatially homogeneous and isotropic. The typical size of a galaxy is $\sim 10^5$ light years, an order of magnitude beyond which there is no preferred position or direction in the universe.

(3) Hydrodynamic approximation : According to which the basic building blocks of the universe are galaxies and their distribution can be considered as a fluid distribution.

The most general metric satisfying the homogeneity and isotropy of the universe is given

by

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{1 - kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \ , \tag{1.1}$$

where t is the time and r, θ, ϕ are space co-ordinates, $a(t)$ is the scale factor of the universe

which gives the expansion history of the universe and k is called the curvature index. This

metric is called the Friedmann-Robertson-Walker (FRW) metric.

   With the hydrodynamic approximation that the matter distribution of the universe can be

approximated to be a perfect fluid, the energy momentum tensor for a perfect fluid distribution

is taken as

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu - pg_{\mu\nu} \ , \tag{1.2}$$

where v_μ's are the components of the fluid velocity vector, ρ is the energy density and p is the

isotropic pressure of the perfect fluid.

With this input, the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.3}$$

lead to the differential field equations

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = 8\pi G\rho, \tag{1.4}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -8\pi Gp, \tag{1.5}$$

where a dot denotes differentiation w.r.t. the cosmic time t.

Also we obtain a third equation, called the matter conservation equation, of the form

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \ , \tag{1.6}$$

which is not an independent equation but can be derived from the two field equations or from

the Bianchi identities. It deserves mention that the curvature index k can not be determined

from the field equations, and is rather put in by hand and can take values 0, +1 and -1 which correspond to flat, closed and open universes respectively.

This system of equations can not be solved completely as there are three unknowns a, ρ and p and only two independent equations. So there is the need of a third equation for solving the system completely which is provided by the equation of state connecting the density and pressure of the cosmic fluid as

$$w = \frac{p}{\rho} \ . \tag{1.7}$$

Normally this equation of state is taken as that of a barotropic fluid, i.e, w is a constant. Thus, at present when the universe is expected to be matter dominated where there is no pressure $(p = 0)$, $w = 0$. Similarly, at very early epoch, when the universe was very hot and was dominated by radiation, then $p = \frac{1}{3}\rho$, i.e, $w = \frac{1}{3}$. So, from these set of equations, $a(t)$ can be solved for and thus one can get an idea about the evolution of the universe.

To relate this FRW model with the observations, some useful parameters are defined.

(i) Hubble constant H is defined as

$$H = \frac{\dot{a}}{a} \tag{1.8}$$

which is an observable parameter and gives the expansion rate of the universe. This parameter has the dimension of $(\text{time})^{-1}$.

(ii) Although the universe is expanding, it is dominated by the gravitational interaction which gives rise to an attractive force. So this expansion is expected to be decelerated. To express this deceleration, a dimensionless deceleration parameter q is defined as

$$q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2} \ ; \tag{1.9}$$

$q > 0$ indicates that $\frac{\ddot{a}}{a}$ is negative, i.e, universe is decelerating and $q < 0$ indicates that $\frac{\ddot{a}}{a}$ is positive, i.e, universe is accelerating.

(iii) A jerk parameter r is defined as

$$r = \frac{\dddot{a}/a}{\dot{a}^3/a^3} , \qquad (1.10)$$

which is a measure of the rate of change of q. As present observations facilitate the study of the evolution of the deceleration parameter q, this jerk parameter has become useful. Sahni et al [11] introduced a pair of "*statefinder parameters*" to characterize the quintessence models, particularly the interacting models. In addition to r, the other parameter s is defined as

$$s = \frac{r - 1}{3(q - \frac{1}{2})} . \qquad (1.11)$$

(iv) The density of the universe is expressed in a dimensionless form by defining a density parameter given by

$$\Omega = \frac{\rho}{\rho_c} , \qquad (1.12)$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is called the critical density or closure density of the universe.

In terms of these parameters, equations (1.4) and (1.5) can be expressed as

$$1 + \frac{k}{a^2 H^2} = \frac{\rho}{\rho_c} = \Omega , \qquad (1.13)$$

$$H^2(1 - 2q) + \frac{k}{a^2} = -8\pi G p . \qquad (1.14)$$

Now, if one considers a matter dominated universe where $p = 0$, equations (1.13) and (1.14) leads to three possibilities :

(a) When $k = -1$, $q < \frac{1}{2}$ and $\Omega < 1$, i.e, $\rho < \rho_c$. This means the matter density being less than the critical density, the universe will go on expanding for ever. Such models are called
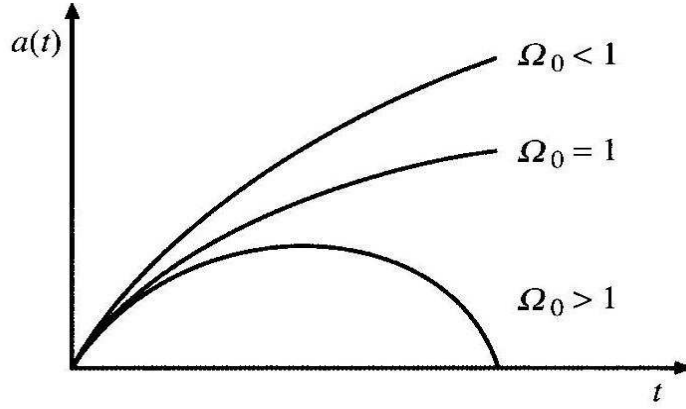
Figure 1.1: *Evolution of the scale factor with time for open, flat and closed models*

open universe models.

(b) When $k = +1$, $q > \frac{1}{2}$ and $\Omega > 1$, i.e, $\rho > \rho_c$. So, the matter density being higher than the critical density, the universe will expand upto some maximum volume and then due to gravitational attraction it will re-collapse. Such models of the universe are called closed universe models.

(c) When $k = 0$, $q = \frac{1}{2}$ and $\Omega = 1$, i.e, $\rho = \rho_c$, one has the limiting case between the above two. This is called a flat model as the space section has zero curvature.

In 1929, Hubble made a remarkable discovery regarding the motion of the galaxies. He observed that the galaxies are moving away from each other, and the velocity of separation v between two galaxies is proportional to the distance D between them, i.e,

$$v \propto D \ ,$$

$$or, \ v = HD \ , \tag{1.15}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble constant defined earlier. This means the galaxies were closer

together earlier. So, by tracking back one can arrive at a time when all the matter were concentrated at a single point. At that instant universe had a zero volume and an infinite density. That epoch, when $a(t) = 0$ and $H \to \infty$ corresponds to some violent activity and is given the name *Big Bang Singularity.* The existence of singularity in a theory is an unwanted feature as laws of physics break down and naturally the features cannot be explained. But still the Big Bang model enjoys the status of a preferred theory as it has its own success stories :

(i) Can predict He abundance : One of the fundamental problems of cosmology is to explain the primary creation of matter and to understand the observed abundances of different elements. At the beginning, when all the matter-energy of the universe was concentrated in a tiny volume, the spectrum of particles that we see today was surely absent. Although the singular stage is definitely out of our purview of explanation, the Big Bang theory can in fact trace the history of the universe when its size was around 10^{-33} cm. This is clearly much shorter than the de Broglie wavelength of most of the particles that we see today. Following Gamow's seminal work [12], one can explain the nucleosynthesis process that took place during the radiation dominated era. Particularly the abundance of lighter elements like Helium are well explained in Big Bang theory and the theoretical prediction has a close semblance with the observational results.

(ii) Could predict the relic thermal radiation at $\sim 2.7~^0$K. This Cosmic Microwave Background Radiation (CMBR) was detected later in 1965 by Penzias and Wilson [13]. This detection confirmed the assumption of isotropy of the universe and is considered to be the greatest triumph of the Big Bang theory and is arguably the strongest pillar of modern cosmology.

(iii) Age of the universe : The Friedmann models could provide a formula for the age of the universe which goes as

$$t \sim \frac{2}{3H} \text{ for a flat matter dominated universe.}$$

Thus knowing the value of H_0 (the subscript '0' indicates the present time), t_0 can be easily calculated. Following this method, the standard Big Bang theory predicts the correct order of magnitude of the age of the universe $\sim 1.5 \ X \ 10^{10}$ years.

(iv) Formation of galaxies : Although the universe is homogeneous at a large scale, there are clumps (galaxies and clusters of galaxies) around us. So at a smaller scale, there should be inhomogeneity. The universe starts homogeneous, and still looks homogeneous at a scale more than 10^6 light years, but at a smaller scale must have inhomogeneities, i.e, structures like galaxies. In the purview of standard Big Bang cosmology, this can also be explained as it has been shown that if there is some kind of perturbation, it can indeed give rise to some growing mode so that galaxies are formed.

1.2   Problems of Standard Cosmological Model :

Standard Big Bang cosmology has its share of problems too. A few of them are :

(1) Horizon problem : Given the present size of the universe ($\sim 10^{10}$ light years), it is quite possible to consider two far separated points such that there is no causal connection between these two points, i.e, their light cones never intersect even if one traces back to the last scattering surface (LSS) when matter and radiation decoupled and the universe became transparent. But even then the two points carry the same information at present as the universe is homogeneous and isotropic. This is known as the '*horizon problem*'.

(2) Flatness problem : From equation (1.13), one can arrive at a relation

$$\Omega - 1 = \frac{k}{a^2 H^2} \ .$$

If it is considered that the initial conditions including the density parameter Ω were set during the GUT epoch when temperature of the universe was $\sim 10^{15}$ GeV, then

$$\Omega = 1 \pm \delta, \text{ where } \delta < 10^{-50} \; .$$

This means the departure from the $\Omega = 1$ value has to be very small. Any relaxation from this fine-tuning would have led to a much higher (or lower) value of Ω at present which is not obtained observationally.

This fine-tuned value of $\Omega \approx 1$ leads to a $k = 0$ model, i.e, a spatially flat model of the universe. Without this extreme fine tuning, the universe would have either collapsed back within a time scale of 10^{-35} s (in a closed model) or would have expanded at a much higher rate (in an open model) than observed at present. Standard Big Bang model can not explain why Ω is so closely tuned to 1 and the problem is termed the '*flatness problem*' or equivalently the '*fine tuning problem*'.

(3) The monopole problem : Gauge field theories suggest that whenever there is any symmetry breaking, inevitably some particles are created which have the characteristics of magnetic monopoles. So, it is expected that during the phase transition of the universe, some monopoles must have been created which being highly stable particles, should have been observed at present epoch also. But, in practice monopoles are not observed. This is known as the '*monopole problem*'.

1.3   The Inflationary Paradigm :

The solution to this problem was suggested by Alan Guth in 1981 [14] by introducing the so called *inflationary model* of the universe. In this model, Guth suggested that during a

very early epoch the universe had a very rapid phase of accelerated expansion having quite a number of e-foldings of the volume in a short span of time. At that time universe was dominated by vacuum energy and the equation of state was of the form

$$\rho_{vac} + p_{vac} = 0 \ ,$$

where ρ_{vac} is the vacuum energy density and p_{vac} is the corresponding pressure.

Now, from the conservation equation (1.6), one obtains $\rho_{vac} = -p_{vac} =$ constant. Thus $\Lambda = 8\pi G \rho_{vac}$ serves as an effective cosmological constant. Then Einstein field equations (1.4) and (1.5) can be written as

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = \Lambda, \tag{1.16}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = \Lambda \ . \tag{1.17}$$

Equations (1.16) and (1.17) can be combined to yield

$$\ddot{a} = \tfrac{\Lambda}{3}a \ .$$

For a positive Λ, $\ddot{a}$ is thus positive and the universe has an accelerated expansion which is termed as the inflationary scenario. The deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$ is negative definite. From equation (1.13) one can write

$$\dot{\Omega} = 2qH(\Omega - 1) \ , \tag{1.18}$$

which clearly indicates that at $\Omega = 1$, one has $\dot{\Omega} = 0$, i.e, $\Omega = 1$ is the stable solution. For a negative value of q, $\dot{\Omega} < 0$ for $\Omega > 1$ and $\dot{\Omega} > 0$ for $\Omega < 1$. So the density parameter decreases or increases to the stable value of $\Omega = 1$ when it is greater or less respectively than $\Omega = 1$. Hence the contribution from the spatial curvature, $\Omega_k = -\frac{k}{a^2 H^2}$, is washed out in view of equation (1.18). Hence, flatness problem was solved.

The horizon problem was also solved in the following way. Two points, which were causally connected during a very early epoch, might have fallen so much apart during the inflationary expansion that at this epoch their past light cones do not have any intersection even if they are extended back to the last scattering surface.

Inflationary models could provide solution to the monopole problem also by considering that the monopoles, that were created during the symmetry breaking, were so diluted during this rapid expansion that the monopole density becomes hardly traceable at present.

So, it is seen that inflationary models could solve the problems of standard Big Bang cosmology more or less satisfactorily. But these models also suffer from some problems, the most famous one being the "*graceful exit problem*". The problem originates from the fact that we see the galaxies and other structures around us. The universe must have a decelerated expansion at some epoch of time in order to facilitate the galaxy formation. This demands that the universe has to come out of this inflationary phase. This problem that how the universe comes out of this rapid expansion phase and enters a decelerated phase of expansion is termed as the "graceful exit problem". A number of models have been suggested for the solution of this problem. All of them have their own merits and pitfalls the details of which are not discussed over here.

So all the major problems in Standard Big Bang Cosmology were believed to be related to the early phase of the history of the universe. The present state of affairs in the universe were presumably competently taken care of, excepting some finer details like that regarding the structure formations. The recent observation that the present universe is accelerating came as jolt and thus the late time behaviour also warrants serious theoretical attention.

1.4 Observational Evidence of the Present

Acceleration :

The evidence in support of an accelerating universe stems from the observations of the luminosity-redshift relation of type Ia supernovae.
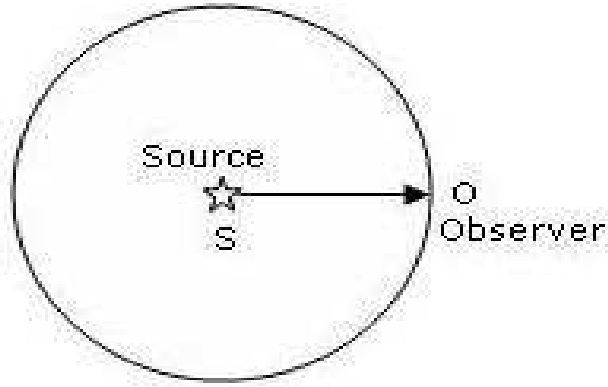
Figure 1.2: *Measurement of luminosity distance d_L from absolute and apparent luminosities*

In a static universe, if one considers a luminous object S emitting a total power L (also called absolute luminosity), then the intensity l (called apparent luminosity) detected by an observer at O at a radial distance d_l from the luminous object (as shown in Figure 1.2) is given by

$$l = \frac{L}{4\pi d_l{}^2} \ .$$
(1.19)

The quantity

$$d_l = \sqrt{\frac{L}{4\pi l}}$$
(1.20)

is known as the luminosity distance. In a static universe, the luminosity distance is equal to the actual distance. In an expanding universe however, the intensity detected by the observer

gets reduced because the energy of a photon emitted gets redshifted due to the cosmological expansion [15]. Because of this expansion, the detected energy gets reduced by a factor of

$$\frac{a(t_0)}{a(t)} = 1 + z \tag{1.21}$$

where $a(t)$ is the scale factor of the universe at some cosmic time t, t_0 is the present time and z is the redshift parameter , given by $z = \frac{\Delta\lambda}{\lambda}$, where λ is the emitted wavelength and the wavelength received is $\lambda + \Delta\lambda$.

Thus in an expanding background, the observed apparent luminosity can be written as

$$l = \frac{L}{4\pi a(t_0)^2 x(z)^2 (1 + z)^2} \tag{1.22}$$

where $x(z)$ is the comoving distance of the luminous object, expressed as a function of the redshift z. This implies that in an expanding universe, the luminosity distance $d_L(z)$ is related to the comoving distance $x(z)$ as

$$d_L(z) = x(z)\,(1 + z) \tag{1.23}$$

where a is normalized such that $a(t_0) = 1$.

As the light geodesics in a spatially flat expanding universe obey the relation

$$cdt = a(z)\,dx(z)\,, \tag{1.24}$$

one can eliminate $x(z)$ using equation (1.23) and express the expansion rate of the universe $H = \frac{\dot{a}}{a}$ in terms of $d_L(z)$ as

$$H(z) = c\frac{1}{\frac{d}{dz}\left(\frac{d_L(z)}{(1+z)}\right)}\,. \tag{1.25}$$

Thus if the absolute luminosity of a distant object is known, its apparent luminosity can be measured as a function of z and from equation (1.20), the luminosity distance d_L can be

calculated as a function of redshift z. The expansion history $H(z)$ can then be deduced by differentiating equation (1.25) with respect to z. On the other hand, if a theoretically predicted $H(z)$ is given, the corresponding $d_L(z)$ can be predicted by integrating equation (1.25) as

$$d_L(z) = c(1+z) \int_0^z \frac{dz'}{H(z')} \ . \qquad (1.26)$$

Then this predicted $d_L(z)$ is compared with the measured $d_L(z)$ to test the consistency of a model. In practice, the astronomers do not use the ratio of absolute over apparent luminosity. Instead they use the difference between apparent magnitude m and absolute magnitude M given by the relation

$$m - M = 2.5 log_{10} \left(\frac{L}{l} \right) \ . \qquad (1.27)$$

It is important to see how the deceleration parameter q, defined by equation (1.9), can be estimated from this observed luminosity at a given redshift. For small values of the time scale and the distance compared to the respective Hubble scales, the scale factor $a(t)$ and hence z can be written as power series,

$$z = H_0(t_0 - t_1) + \left(1 + \frac{q_0}{2} \right) H_0^2(t_0 - t_1)^2 + \ .... \ , \qquad (1.28)$$

where the quantities with a suffix zero indicate their present values. Using this expression, the luminosity distance d_L can be written as

$$d_L = H_0^{-1} \left[z + \frac{1}{2}(1 - q_0)z^2 + \ .... \right] \ , \qquad (1.29)$$

and hence the apparent luminosity l becomes

$$l = \frac{L}{4\pi d_l^2} = \frac{LH_0^2}{4\pi z^2} [1 + (q_0 - 1)z + \ ....] \ . \qquad (1.30)$$

The apparent and absolute magnitudes will then be related as

$$m - M = 25 - 5 log_{10} H_0 \ (km/sec/Mpc) + 5 log_{10} cz \ (km/sec) + 1.086(1 - q_0)z + ... \qquad (1.31)$$

The apparent luminosity is smaller for a negative q_0 and thus the dimmer appearance of the supernovae calls for a negative q_0. The expansion history of the universe is very well depicted
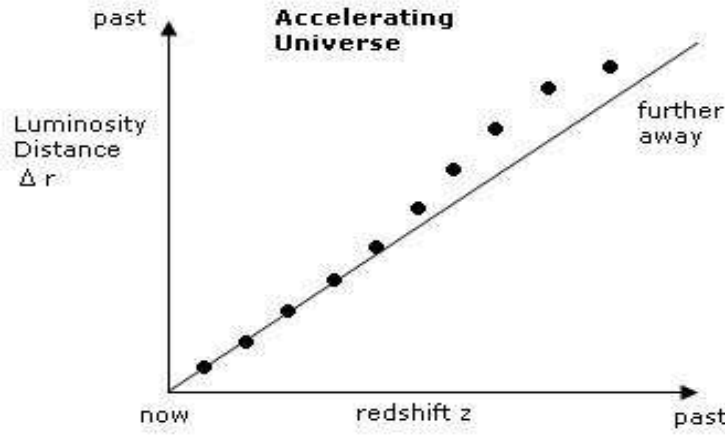
Figure 1.3: *Hubble diagram for an accelerating universe*

by the Hubble diagram where the x-axis shows the redshift z of a luminous object and the y-axis shows the physical distance Δr to those objects. This Δr is in fact the luminosity distance. As the redshift z is related to the scale factor $a(t)$ at the time of emission of radiation via equation (1.21), whereas Δr is related to the time in past when the emission was made, therefore the Hubble diagram provides information about the time dependence of the scale factor $a(t)$.

The slope of this diagram at a given redshift denotes the inverse of the rate of expansion $H(z)$, i.e,

$$\Delta r = \frac{1}{H(z)} c\, z \; . \tag{1.32}$$

In an accelerating universe, the slope H^{-1} of the Δr vs. z curve is larger at high redshift. Thus, at a given redshift, luminous objects appear to be at a greater distance, i.e, dimmer as compared to an empty universe expanding with a constant rate (see Figure 1.3).

In the construction of Hubble diagram, those luminous objects are used whose absolute luminosity is known and therefore by measuring their apparent luminosity, their distances can be calculated. Those luminous objects are called standard candles or distance indicators. Type Ia supernovae serve as excellent standard candles for estimating the luminosity distance d_L. Type Ia supernovae are explosions believed to occur in binary star systems where one of the companions has a mass below the Chandrasekhar limit $1.4 M_\odot$ and thus become a white dwarf supported by degenerate electron pressure after hydrogen and helium contained in the star are burnt up. Once the other companion reaches the red giant phase, the white dwarf starts accreting matter from the companion star. Once the mass of the white dwarf star becomes equal to the Chandrasekhar limit, the gravitational pull overcomes the degeneracy pressure and the white dwarf starts to shrink. This increases the temperature and results in the carbon fusion. This leads to violent explosions, commonly called supernovae explosions. These explosions are detected by a light curve whose luminosity increases rapidly in a time scale of less than a month, reaches a maximum and disappears in a timescale of 1-2 months (see figure 1.4). Type Ia supernovae are characterised by the absence of hydrogen and abundance of silicon in the spectrum.

Supernovae are preferred standard candles mainly for the following reasons :

i) These objects are highly luminous. This high absolute luminosity of supernovae Ia ($M = -19.5$) ensures that they can be seen from large distances (~ 1000 Mpc) and thus are useful for measuring various cosmological parameters.

ii) The dispersion in supernovae luminosity at maximum light is extremely small and the corresponding change in intensity is $\sim 25\%$.

iii) Their explosion mechanism is fairly uniform and well understood.
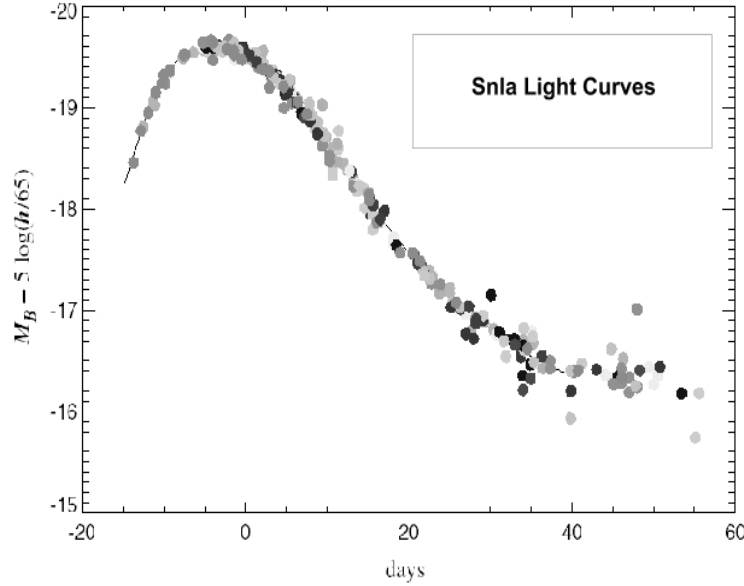
Figure 1.4: *Light curve for a typical Supernova Ia*

However, the major problem in using Sn Ia as standard candles is that they are rare events - for instance, in our galaxy they occur only a few times in a millennium. Also it is not easy to predict a supernovae explosion. However, the key feature of the high precision tools of observation is that one can now detect supernovae in other galaxies also. The basic strategy employed in observing supernovae Ia is as follows [16, 17, 18, 19, 20, 21] :

i) A number of wide fields of apparently empty sky are observed. With modern instruments on a 4 meter-class telescope, tens or thousands of galaxies are observed in a few patches of sky.

ii) three weeks later, the same galaxies are observed once again.

iii) The images are then subtracted to observe the supernovae explosions.

The result of this observation strategy is a set of Sn Ia light curves in various bands of spectrum.

From the light curves, their peak apparent luminosity is used to construct the Hubble diagram.

The first project in which supernovae were used to determine the energy associated with the cosmological constant was carried out by Perlmutter et al. in 1997 [22]. This project was named as Supernovae Cosmology Project (SCP). In one year they discovered seven supernovae at redshift $0.35 < z < 0.65$ and observed them with different telescopes from the earth. The Hubble diagrams they constructed were in good agreement with a standard decelerating Friedmann cosmology. However, one year later they updated their results by including the measurements of a very high redshift ($z \sim 0.83$) Supernovae Ia [16]. This dramatically changed the scenario and a decelerating universe was ruled out at about 99% confidence level. This result was confirmed independently by another pioneer group, viz, High-z Supernovae Search Team (HSST) [17]. They had discovered 16 supernovae at a redshift $0.16 < z < 0.62$ and the results indicated an accelerating universe at a 99% confidence level.

In 2003, Tonry et al. [18] reported the results of eight newly discovered supernovae in the range $0.3 < z < 1.2$. Their results reinforced the previous findings of accelerated expansion and also gave the confirmation of decelerated expansion at $z > 0.6$. So, obviously the universe must have had a transition from deceleration to acceleration in the past. This transition was confirmed and pinpointed by Riess et al. in 2004 [13]. They included 16 new high redshift supernovae and after analyzing all available data, constructed a reliable and robust data set consisting of 157 points which is known as Gold data set. With this new data set, they could clearly identify the transition from decelerated to accelerated expansion at $z \approx 0.46 \pm 0.13$. However, it was not easy to conclude whether the data favour an accelerating or decelerating universe from only the Hubble diagram corresponding to the Gold data set because of the error bars present in the data set. This would be easier if the Hubble diagram of figure 1.3 where the

distance is plotted vs redshift is superposed with the distance-redshift relation $d_L^{empty}(z)$ of an empty universe with $H(z)$ constant. So, a more efficient plot was used for this purpose, viz, the logarithmic plot of $d_L(z)$ vs. $d_L^{empty}(z)$ which could easily distinguish between an accelerated and decelerated expansion. Such a plot for the Gold data set clearly indicated that the best fit is obtained by an expansion which was decelerated at the earlier times ($z > 0.5$) and accelerated at recent times ($z < 0.5$). Attempts have been made to explain the observed dimming of supernovae at high redshift by considering that this apparent dimming is due to the scattering of light by intergalactic dust or grey dust or even due to evolution of Sn Ia. However none of them has a firm footing [13, 24, 25] and thus strengthens the belief that the universe at present is undergoing an accelerated phase of expansion. This is also confirmed by the highly accurate Wilkinson Microwave Anisotropy Probe (WMAP) data [5, 6, 7, 8, 9].

1.5   Search for the Dark Energy :

This observed acceleration of the universe brings in trouble as gravity is attractive and this acceleration can not be driven by the attractive gravitational properties of regular matter. So, obviously an additional component is required which can give rise to an effective repulsive gravity so that matter can move away from each other with an acceleration. The obvious question to address is therefore, "*What should be the properties of this additional component?*" The answer can be obtained by comparing Newtonian gravity with Einstein's gravity.

In Newtonian gravity, the acceleration of a test particle having mass m under the influence of gravity is given by

$$m\ddot{a} = -\frac{GMm}{a^2}$$

which gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho \tag{1.33}$$

as $M = \frac{4}{3}\pi a^3 \rho$.

On the other hand, in Einstein's gravity, from equations (1.4) and (1.5) one can obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \ . \tag{1.34}$$

This can be written in terms of equation of state parameter w as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho(1 + 3w) \tag{1.35}$$

which gives a measure of the acceleration of the universe.

So, from equations (1.33) and (1.34), it is evident that unlike that in Newtonian gravity, in Einstein gravity, pressure also plays a major role along with the density in determining the space-time dynamics of the universe. In other words, *"pressure carries weight in Einstein's gravity"*.

Now, as the universe at present is accelerating, $\ddot{a} > 0$. From equation (1.34), a positive $\ddot{a}$ can be obtained only if p is sufficiently negative such that $\rho + 3p < 0$ or equivalently $w < -\frac{1}{3}$. So in order to explain the observed acceleration of the universe there is indeed the requirement of some form of matter which can generate sufficient negative pressure. This particular form of matter, now popularly referred to as *"dark energy"*, is believed to account for as much as 70% of the present energy of the universe.

A large number of possible candidates suitable as dark energy component have appeared in the literature. None of them has a clear advantage over all others. All have their merits, but none perhaps has a firm theoretical footing. Its effect has been, as discussed, is to provide a sufficient effective negative pressure. Nothing is known about its distribution vis-a-vis the

dark matter, except that it does not cluster at any scale lower than the size of the universe. A few of these candidates are described below.

1.5.1 Cosmological Constant Models :-

The simplest dark energy candidate is the cosmological constant Λ introduced by Einstein in 1917. With the introduction of the Λ-term, the Einstein's equation gets modified as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \ . \tag{1.36}$$

Einstein originally introduced the Λ-term on the left hand side of the field equation in order to obtain a static universe. But later on, when Hubble's observations suggested that the universe is expanding, he himself rejected the Λ-term. However, Λ was again brought into being in early 1980's with the inflationary model of the universe [14]. During inflation, the universe was dominated by the vacuum energy ρ_{vac} as discussed earlier. As the equation of state for such energy is

$$\rho_{vac} + p_{vac} = 0,$$

both ρ_{vac} and p_{vac} are constants, and the Einstein field equations will effectively look like

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = \Lambda, \tag{1.37}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = \Lambda \ , \tag{1.38}$$

where $\rho_{vac} = -p_{vac} = \frac{\Lambda}{8\pi G}$.

Here Λ is not introduced arbitrarily, but rather attains the significance of the vacuum energy density. By solving these equations one gets an exponentially expanding model of the universe at a very early epoch which provides solutions to the problems of Big Bang theory. As the

simple inflationary model had its own problems, such as that of the 'graceful exit', or the huge discrepancy in the theoretically predicted value of Λ and the one suggested by observations, other more complicated models took over and Λ had to hide itself into oblivion. In the late 90's, with the observation that universe is at present accelerating, Λ again came back strongly after a brief period of hibernation.

For an FRW universe, the Einstein's equations with a cosmological constant take the form

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = 8\pi G\rho + \Lambda, \tag{1.39}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -8\pi Gp + \Lambda . \tag{1.40}$$

From equations (1.39) and (1.40), one easily arrives at the relation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} , \tag{1.41}$$

from which it is clear that Λ is capable of providing an acceleration ($\ddot{a} > 0$) if

$$\Lambda > 4\pi G(\rho + 3p) .$$

In a very simple case, when the universe is spatially flat ($k = 0$) and is dust dominated ($p = 0$), equations (1.39) and (1.40) gives an exact analytic expression for the scale factor as

$$a(t) \propto \left(\sinh\frac{3}{2}\sqrt{\frac{\Lambda}{3}}t\right)^{2/3} . \tag{1.42}$$

For a very small t (early epoch), $a(t) \propto t^{2/3}$ and hence $q = \frac{1}{2}$, which gives a decelerated expansion in the early phase of dust dominated era as expected [26]. On the other hand, for large t (i.e, present epoch), $a \propto e^{\sqrt{\frac{\Lambda}{3}}t}$ which gives an accelerated expansion. This model is popularly known as ΛCDM model. However, there is a major problem related to the cosmological constant Λ. Field theory suggests that the lower limit of the value of vacuum energy density should be

$$\rho_{vac} \approx 10^6 \; GeV^4 \; ,$$

whereas the current observational upper limit on the cosmological constant is

$$\rho_{vac}|_0 \approx 10^{-29} \; g/cm^3 \approx 10^{-47} \; GeV^4 \; ,$$

where the subscript '0' stands for present time. So, there is a discrepancy between the predicted value and the observed value by 10^{53} orders of magnitude. The cosmological constant Λ is expected to have a large value during early epoch so as to resolve - via inflation - the horizon and flatness problems; at the same time it requires a low value at the present epoch to avoid conflict with the observations. This problem is referred to as the "*cosmological constant problem*" because of which Λ has lost some of its ground.

To avoid this problem, some dynamical models of dark energy have been proposed where Λ, instead of being a constant, is a slowly varying function of time.

1.5.2   Varying Λ Models :-

The first candidate for a dynamical model of dark energy was the time varying cosmological parameter $\Lambda(t)$. A number of phenomenological models have been described to introduce this dynamical Λ-term as the new form of matter [27, 28, 29, 30, 31, 32] and is taken as a function of the scale factor $a(t)$ or the cosmic time t. However, there is no strong physical motivation behind this choice. In fact, the dynamical Λ-term leads to some problems. If Λ is considered as a constant, then the introduction of the term $\Lambda g_{\mu\nu}$ in the Einstein field equations does not affect the conservation equation. It is so because the conservation equations come as a consequence of Bianchi identity

$$G^{\mu\nu}_{;\nu} = 0 \; ,$$

which remains the same as for a constant Λ,

$$(\Lambda g^{\mu\nu})_{;\nu} = \Lambda_{,\nu} g^{\mu\nu} + \Lambda g^{\mu\nu}_{;\nu} = 0 \ .$$

However, in a varying Λ model, the Bianchi identity leads to a modified conservation equation. If the normal matter is allowed to satisfy its own conservation equation of the form given in equation (1.6), Λ automatically turns out to be a constant. The way out is to consider some interaction between $\Lambda(t)$ and matter such that one grows at the expense of the other. But in order to incorporate this interaction, one has to sacrifice the matter conservation equation. In most of the cases, the choice of the form of interaction does not have any physical background. So, the second natural choice for a dynamical Λ-term is a scalar field ϕ having some potential $V(\phi)$ and is popularly called the *"quintessence scalar field"*.

1.5.3 Quintessence Models :-

Inspired by the role of a scalar field with a suitable potential in providing an inflationary scenario in the early universe, attempts are made to formulate a "dynamical dark energy" model based on scalar field cosmology. With the introduction of a scalar field minimally coupled to gravity, the action gets modified as

$$S = \int d^4 x \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) + L_m \right] \ . \tag{1.43}$$

For a spatially flat FRW cosmology, Einstein's field equations take the form

$$3\frac{\dot{a}^2}{a^2} = \rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi) \ , \tag{1.44}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -p_m - \frac{1}{2}\dot{\phi}^2 + V(\phi) \ . \tag{1.45}$$

Variation of the action with respect to the scalar field ϕ leads to the wave equation for ϕ as

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'(\phi) = 0 \tag{1.46}$$

where a prime indicates differentiation with respect to ϕ.

In view of the equations (1.44) - (1.46), the matter conservation equation

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 0 \tag{1.47}$$

is not an independent equation, and comes as a consequence of the Bianchi identities. Thus one is left with five unknowns, a, ρ_m, p_m, ϕ and $V(\phi)$ and only three independent equations to solve them. If the equation of state $p_m = p_m(\rho_m)$ and the form of potential $V = V(\phi)$ are given, the system of equations is closed. Equations (1.44) and (1.45) reveal that the contribution to the effective energy density and pressure from the scalar field is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) , \tag{1.48}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) , \tag{1.49}$$

where the first term on the right hand side represents the kinetic part whereas the second term represents the potential part.

In the matter dominated era the fluid pressure $p_m = 0$, and equation (1.34) yields the result

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_m + \rho_\phi + 3p_\phi]$$

$$= -\frac{4\pi G}{3}\left[\rho_m + 2\dot{\phi}^2 - 2V(\phi)\right] . \tag{1.50}$$

If $V(\phi)$ grows to a sufficiently large value such that

$$2V(\phi) > \rho_m + 2\dot{\phi}^2 , \tag{1.51}$$

$\ddot{a}/a$ has a positive value consistent with the present observation. These models have a further interesting possibility that if $V(\phi)$ has a smaller value compared to $\frac{\rho_m + 2\dot{\phi}^2}{2}$ in the early matter dominated era and grows later to dominate the dynamics, the model would exhibit a signature flip in q from a positive to a negative value at a recent past. A fine tuning of the parameters of the theory and the arbitrary constants of integration might lead to models consistent with the details of observational data. These features have made scalar field cosmologies a very active area of research, and now the scalar field with a potential giving rise to a negative pressure is specifically called a "*quintessence field*" and the generic term for any field, that drives the late surge of accelerated expansion of the universe, has become '*dark energy*'.

The system of equations suggest that if a particular dynamics of the universe, i.e, the temporal behaviour of the scale factor $a(t)$ is chosen, it is possible to find a potential consistent with that. Amongst the host of quintessence potentials available in the literature, some give an ever accelerating model, some give an acceleration in the large t limit, and a few indeed allows for a smooth transition from a decelerated to an accelerated expansion in the matter dominated era itself consistent with the observation that the universe has entered the accelerated phase of expansion in the late matter dominated era.

One typical example of the latter kind of a quintessence field is the one introduced by Sen and Sethi [33], where the potential is a double exponential, given by

$$V(\phi) = \alpha(e^{2\beta\phi} + e^{-2\beta\phi}) + V_0 \ . \tag{1.52}$$

where α, β and V_0 are constants. The solution of the scale factor comes as $\sinh^{\beta}(t/t_0)$. For a small t, the solution for the scale factor is like $\sim t^{\beta}$ whereas for a high value of t, $a \sim e^{\beta t/t_0}$. For $\beta < 1$, the model has a decelerated expansion in the early epoch, but indeed accelerates at a matured age. Similar kind of potentials have been used by Barreiro et al [34] and Rubano

et al [35] as well.

A particularly interesting class of quintessence fields are the so called *"tracker fields"* for which the potential is steep enough to satisfy the condition

$$\Gamma = \left(\frac{V''}{V}\right) / \left(\frac{V'^2}{V^2}\right) \geq 1 \ . \tag{1.53}$$

All potentials satisfying this condition lead to a common evolutionary path for the fields starting from a wide range of initial conditions [36]. The energy density of the tracker fields has an evolution which mimics that of the background matter during most of its history (see Figure 1.5) and dominates over the matter density at later stages of the evolution to generate acceleration. Thus, these models can alleviate the coincidence problem which poses the question why the dark energy sector should dominate the dynamics of the universe at present.
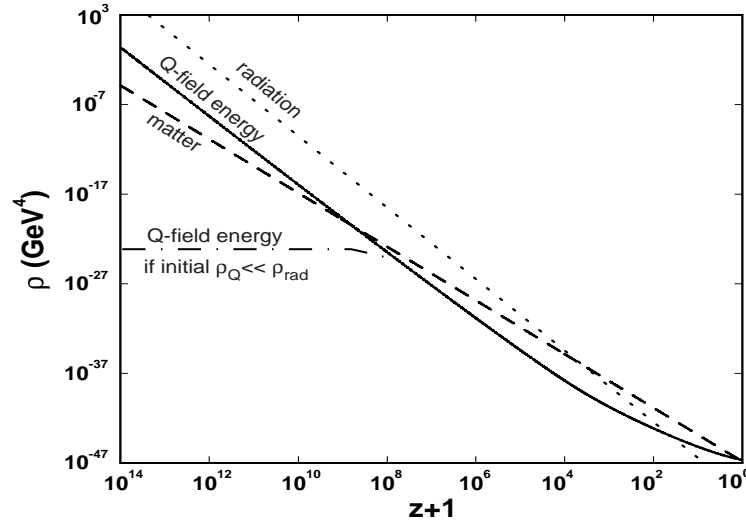
Figure 1.5: *The quintessence field rolling down an inverse power law potential tracks first radiation, then matter and finally dominates the energy density of the universe at present. (From Zlatev, Wang and Steinhardt [36])*

Some simple examples are the inverse power law potential [37]

$$V(\phi) = \frac{V_0}{\phi^\alpha},$$ (1.54)

with some restriction on α, and the exponential potential [37, 38, 39]

$$V(\phi) = V_0 \, e^{-\lambda\phi} \, .$$ (1.55)

Within the tracker framework, a potential with a versatile ambition was given by Sahni and Wang [40]. This potential is given as

$$V(\phi) = V_0[\cosh(\lambda\phi) - 1]^p \, .$$ (1.56)

For $|\lambda\phi| >> 1$ and $\phi < 0$, the potential is exponential,

$$V(\phi) \propto \exp(-p\lambda\phi) \, ,$$

and $w_\phi = \frac{p_\phi}{\rho_\phi}$ is very close to $w_m = \frac{p_m}{\rho_m}$.

On the other hand, when $|\lambda\phi| << 1, V(\phi) \propto (\lambda\phi)^{2p}$. Then, the average equation of state parameter is given by

$$< w_\phi > = \frac{p-1}{p+1}.$$

For $p \leq \frac{1}{2}$, the model describes quintessence whereas for $p = 1$ it can play the role of a cold dark matter. So, this model is able to describe both dark matter and dark energy within a tracker framework (see [41, 42]).

Another useful potential with interesting features was proposed by Zlatev et al.[36] as

$$V(\phi) = V_0 \left[e^{M_p/\phi} - 1\right] \, .$$ (1.57)

The advantage of this potential is that it can significantly alleviate the fine tuning problem and ρ_ϕ can come to dominate over the present matter density from a large number of initial conditions.

There are of course a lot of other examples. However, despite the many attractive features of these quintessence potentials, a degree of fine tuning does remain in fixing the parameters of the potentials. The generic problem of quintessence models is that the potentials are all taken arbitrarily and none of them has a sound physical basis. But indeed if we know the kind of acceleration required, more often than not, one can find out the form of potential which generates the desired acceleration.

1.5.4   Non-minimally Coupled Scalar Field Models :-

In these models, the scalar field is non-minimally coupled to gravity such that the general action is of the form

$$S = \int d^4x \sqrt{-g} \left[\frac{f(\phi)R}{16\pi G_0} - g(\phi)\phi^{,\mu}\phi_{,\mu} + L_m \right] , \tag{1.58}$$

where $f(\phi)$ and $g(\phi)$ are some functions of the scalar field ϕ, G_0 is the Newtonian constant of gravity.

Comparing this action with the Einstein's action, one gets

$$G \sim \tfrac{1}{f(\phi)} .$$

So, in this type of scalar field models, G is not a constant but is some function of the scalar field ϕ.

The simplest of the non-minimally coupled scalar field models is the Brans-Dicke theory. This theory was first introduced to incorporate Mach's principle in a relativistic theory of

gravity. According to this principle, the inertia of a body is not an intrinsic property of its own, it rather depends on the mass distribution of the rest of the universe.

In Brans-Dicke theory

$$f(\phi) = \phi ;$$

$$g(\phi) = \frac{\omega}{\phi} , \tag{1.59}$$

ω being a dimensionless constant parameter. With these, the action for the Brans-Dicke theory takes the form [43]

$$S = \int d^4x \sqrt{-g} \left[\frac{\phi R}{16\pi G_0} - \frac{\omega}{\phi} \phi^{,\mu} \phi_{,\mu} + L_m \right] . \tag{1.60}$$

If we consider the matter field to consist of a perfect fluid, then the field equations for a spatially flat Robertson - Walker spacetime become,

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_0 \frac{\rho_m}{\phi} + \frac{\omega}{2} \frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} , \tag{1.61}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G_0 \frac{p_m}{\phi} - \frac{\omega}{2} \frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} . \tag{1.62}$$

Also, the wave equation for the Brans-Dicke field is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{8\pi G_0}{(2\omega + 3)} (\rho_m - 3p_m) . \tag{1.63}$$

Dicke [44] in 1962 framed an alternative version of their scalar tensor theory [43] by a simple redefinition of units. By effecting a conformal transformation

$$\bar{g}_{\mu\nu} = \phi g_{\mu\nu} , \tag{1.64}$$

the action given by equation (1.60) becomes,

$$S = \int \left[\frac{\bar{R}}{16\pi G} + \frac{(2\omega + 3)}{2} \Psi_{,\mu} \Psi^{,\mu} + \bar{L}_m \right] , \tag{1.65}$$

where an overhead bar represents quantities in new frame and $\Psi = \ln\phi$.

Comparing this action with equation (1.60), it is seen that in this version ϕ is no longer coupled to $\bar{R}$. So, the effective constant of gravitation G, which is a function of the scalar field as

$$G = \frac{G_0}{\phi} \,, \tag{1.66}$$

in the original version of the theory, now becomes a constant.

The field equations (1.61) and (1.62) in the new frame look like

$$3\frac{\dot{\bar{a}}^2}{\bar{a}^2} = \bar{\rho}_m + \frac{(2\omega + 3)}{4}\dot{\Psi}^2 \,, \tag{1.67}$$

$$2\frac{\ddot{\bar{a}}}{\bar{a}} + \frac{\dot{\bar{a}}^2}{\bar{a}^2} = -\bar{p}_m - \frac{(2\omega + 3)}{4}\dot{\Psi}^2 \,, \tag{1.68}$$

and the wave equation becomes

$$\ddot{\Psi} + 3\frac{\dot{\bar{a}}}{\bar{a}}\dot{\Psi} = \frac{\bar{\rho}_m - 3\bar{p}_m}{(2\omega + 3)} \,, \tag{1.69}$$

where an overhead bar indicates quantities in new frame and $\bar{a}^2 = \phi a^2$. The equations are written in units where $8\pi G_0 = 1$. The density and pressure of the normal matter in this version are related to those in the original version as

$$\bar{\rho}_m = \phi^{-2}\rho_m \ \text{ and } \ \bar{p}_m = \phi^{-2}p_m.$$

The resulting field equations in the new frame look more tractable than the original version. Furthermore, the equations in this version give insight regarding the comparison of the energies of different components of matter. For example, it is clearly seen from equation (1.67) that the contribution to the energy density by the scalar field is given by

$$\bar{\rho}_\phi = \frac{(2\omega+3)}{4}\dot{\Psi}^2.$$

However, one has to pay some price for it. In the transformed version although G becomes a constant, the rest mass of a test particle becomes a function of the scalar field [44] and one has to sacrifice the equivalence principle. So, the geodesic equations are no longer valid and indeed the physical significance of different quantities in this version of the theory is somewhat obscure. However, because of its computational simplicity, it is easier to arrive at some solutions in the transformed version. And the problem can be resolved by transforming back to the original atomic units where one can talk about the various features more confidently.

Although General Relativity (GR) is a better theory of gravity and it enjoys experimental evidences in its support, for various reasons Brans-Dicke (BD) theory continues to enjoy an alive interest. One important advantage of BD theory is that it becomes indistinguishable from GR in the limit $\omega \to \infty$. This limit has now shown to have only a restricted application [45], but the PPN parameters calculated in BD theory [46] clearly shows that at least in the weak field limit the predictions from local astronomical observations in this theory will be same as that in GR in the large ω limit. For these reasons, there is a popular notion that BD theory is perhaps the most natural generalization of GR. However, the local astronomical observations suggest that if BD theory has to be consistent, then $\omega \sim 10^3$ which renders the theory practically indistinguishable from GR [47].

BD theory proved to be useful in providing clues to the solutions for some of the outstanding problems in cosmology. In 1981, Guth [14] proposed the inflationary model of cosmology. This model could provide solution to many of the cosmological problems as discussed earlier, but it suffered from the '*graceful exit problem*'. A large number of models were introduced to solve this problem having their own merits and demerits.

In 1984, Mathiazhagan and Johri [48] addressed the problem in Brans-Dicke (BD) theory [43]. Under this framework, it was shown that along with a vacuum energy, the scale factor grows as a power function of time. Using a similar technique, La and Steinhardt [49] presented the "*extended inflation model*" in order to get a sufficient slow roll of the scalar field so that there is sufficient time for the completion of phase transition and thus the graceful exit problem could be resolved. However, this leads to unacceptable distortions of the microwave background [50]. To solve this problem, Steinhardt and Accetta [51] developed the "*hyper-extended inflation model*". Later Brans-Dicke theory was used for finding a solution to the graceful exit problem with a large number of potentials [52], where the inflaton field oscillates during the later stages of evolution and the universe comes out of the inflationary phase.

BD theory has also found applications in solving a few recent cosmological problems, such as, the quintessence problem. A number of models have been presented where the Brans-Dicke scalar tensor theory could potentially solve the problem of quintessence as it leads to non-decelerating solutions for the scale factor in the present matter dominated universe. In some of these models [53, 54], the BD theory is modified by incorporating a potential $V(\phi)$ which is a function of the BD scalar field itself which could drive the acceleration. However, the problem with these models is that $V(\phi)$ is put in by hand and there is no physical motivation behind the choice of form of $V(\phi)$.

A few other models have also appeared in the literature where the cosmic acceleration is obtained with a quintessence field in BD theory [55]. However, this result hardly provides any improvement on the corresponding GR result. Recently Banerjee and Pavon [56] have proposed a model where the BD theory could explain the present accelerated expansion of the universe without resorting to a cosmological constant or quintessence matter. This is better

in the sense that one does not have to invoke any additional quintessence field to explain the acceleration. However, this model have problems in providing a decelerated expansion in the radiation-dominated epoch.

The general defect of all these models is that none of them could provide a smooth transition from the decelerated to accelerated phase of expansion and they rather provide an acceleration in some limit. Also in all of these models, a consistent accelerated solution is obtained only for small negative values of the Brans-Dicke parameter ω. This is in sharp contrast to the value obtained from local astronomical experiments which predict the value of ω to be of the order of a thousand [47]. Attempts have been made to overcome this problem by considering a modified version of Brans-Dicke theory, called Nordtvedt's theory, where the parameter ω is a function of the Brans-Dicke scalar field instead of being a constant [57]. In this case the field equations (1.61) and (1.62) remain intact, but the wave equation (1.63) gets modified as

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{\rho - 3p}{(2\omega + 3)} - \frac{\dot{\omega}\dot{\phi}}{(2\omega + 3)} \ . \tag{1.70}$$

Bartolo and Pietroni [58] have pointed out that a varying ω can indeed explain the late time behaviour of the universe. Also Banerjee and Pavon [56] showed that a varying ω theory could give rise to a decelerating radiation model followed by an accelerating model in the matter dominated universe. However, none of these could provide smooth transition from deceleration to acceleration in the matter dominated era itself. So, a thorough survey of varying ω theory is indeed warranted to check if it gives rise to a model of the universe which can explain the transition from decelerated to accelerated phase of expansion in the matter dominated epoch itself with some high values of ω consistent with local astronomical experiments.

1.5.5   Curvature Driven Accelerating Models :-

The scalar field models or the cosmological constant models are amongst the most popular candidates of dark energy component. Recently, an attempt along a different direction is also gaining attention. This effort explores the possibility of whether geometry by itself can serve the purpose of providing late time acceleration of the universe.

The idea actually originates from the experience of inflationary models. It was shown by Starobinsky [59] and Kerner et al [60, 61] that higher order modifications of the Ricci curvature R, in the form of R^2 or $R_{\mu\nu}R^{\mu\nu}$ in the Einstein - Hilbert action, could generate sufficient acceleration in the very early universe. However, with the evolution of the universe, R is expected to fall off. This leads to the question whether the inverse powers of R, which becomes dominant during the late time, can help driving the recent acceleration.

The action gets modified as,

$$S = \int \left[\frac{1}{16\pi G} f(R) + L_m \right] \sqrt{-g} d^4 x \tag{1.71}$$

where the usual Einstein - Hilbert action is generalized by replacing R with an arbitrary function $f(R)$. A variation of this action with respect to the metric yields the field equations as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T^c_{\mu\nu} + T^M_{\mu\nu} \,, \tag{1.72}$$

where $T^c_{\mu\nu}$ represents the contribution from the curvature and $T^M_{\mu\nu}$ denotes the energy momentum tensor components for the matter field scaled by a factor of $\frac{1}{f'(R)}$. Here the choice of units $8\pi G = 1$ has been made.

$T^c_{\mu\nu}$ is explicitly given as,

$$T^c_{\mu\nu} = \frac{1}{f'(R)} \left[\frac{1}{2} g_{\mu\nu} (f(R) - R f'(R)) + f'(R)^{;\alpha\beta} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\nu} g_{\alpha\beta}) \right] \,, \tag{1.73}$$

where a prime indicates differentiation with respect to the Ricci scalar R.

For a spatially flat Robertson-Walker spacetime, where

$$ds^2 = dt^2 - a^2(t) \left[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] , \tag{1.74}$$

the field equations (1.72) take the form

$$3\frac{\dot{a}^2}{a^2} = \frac{1}{f'(R)} \left\{ \frac{1}{2}[f(R) - Rf'(R)] - 3\frac{\dot{a}}{a}\dot{R}f''(R) \right\} + \rho_m \tag{1.75}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{f'(R)} \left\{ 2\frac{\dot{a}}{a}\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2 f'''(R) - \frac{1}{2}(f(R) - Rf'(R)) \right\} - p_m . \tag{1.76}$$

It is evident that if $f(R) = R$, the field equations (1.75) and (1.76) take the form of usual Einstein field equations.

The Ricci scalar R is given by

$$R = -6 \left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right] , \tag{1.77}$$

which involves a second order derivative of the scale factor a. As equation (1.76) contains $\dddot{R}$, one actually has a system of fourth order differential equations. Depending on the functional form of $f(R)$, some of the terms on the right hand side of equation (1.76) can provide an effective negative pressure and generate sufficient acceleration.

A substantial amount of work has already been done along this line by choosing various functional forms of $f(R)$. Capozziello et. al [62, 63] considered $f(R) = R^n$ and showed that it leads to an accelerated expansion for $n = -1$ and $n = \frac{3}{2}$. The dynamical behaviour of R^n gravity has been studied in detail by Carloni et. al [7]. Carroll et. al [5] used a combination of R and $\frac{1}{R}$ in the action and a conformally transformed version of the theory where the effect of curvature is formally taken care of by a scalar field having some potential. They showed that it could generate a negative value for the deceleration parameter q. Vollick [6] used $\frac{1}{R}$ term

in the action and obtained an exponentially expanding and hence accelerating model for the universe. It deserves mention that Vollick actually employed a Palatini variation, so the field equations are different from equations (1.75) and (1.76). Nojiri and Odinstov [8] considered the Lagrangian of the form

$$L = R + R^m + R^{-n} \text{ where } m, n \text{ are positive integers,}$$

and showed that it is indeed possible to obtain an inflation at the early stage and a late time accelerated expansion from the same set of field equations. Other interesting investigations include the choice of $f(R)$ as $\sinh^{-1}(R)$ [68] or $\ln R$ [10], which also could provide late time acceleration. However, all these models mentioned above have problems regarding the stability [11]. Furthermore, most of them either resort to a piecewise solution for large R and small R, or provide acceleration in some limit or an eternally accelerating model. But none of them could show the transition from decelerated to accelerated phase of expansion in the same matter dominated regime. But still these investigations open up an interesting possibility for the search of dark energy in the non-linear contributions of the scalar curvature.

1.5.6 Chaplygin Gas Models :-

A *chaplygin gas model* is also one of the important candidates for solving the dark energy problem. The Born-Infeld lagrangian density

$$L = -V_0 \sqrt{1 - \phi_{,\mu} \phi^{,\mu}} \tag{1.78}$$

leads to the chaplygin gas obeying the equation of state

$$p = -\frac{A}{\rho} \tag{1.79}$$

where $A = V_0{}^2$.

The chaplygin gas can also be derived from a quintessence lagrangian

$$L = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

with the potential [71]

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh 3\phi + \frac{1}{\cosh 3\phi}\right).\tag{1.80}$$

The conservation equation

$$d(\rho a^3) + p\, d(a^3) = 0\tag{1.81}$$

immediately gives

$$\rho = \sqrt{A + \frac{B}{a^6}}\tag{1.82}$$

where B is a constant of integration if the equation of state is given by equation (1.79).

The more general form of equation (1.79) leads to the equation of state for the generalized chaplygin gas given by (see [72] and references therein)

$$p = -\frac{A}{\rho^\alpha}\tag{1.83}$$

where $0 < \alpha < 1$ and A is a positive constant. $\alpha = 1$ gives back the old chaplygin gas model. In the framework of FRW cosmology, this equation of state yields solution of the Einstein equations and leads to density evolving as

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}},\tag{1.84}$$

where a is the scale factor of the universe and B is an integration constant. From equations (1.82) and (1.84), it is clear that for small t, i.e, for small a, one can obtain a dust dominated model $\left(\rho \sim \frac{1}{a^3}\right)$ whereas for large t, $\rho \sim \sqrt{A}$ or $(\rho \sim A^{\frac{1}{1+\alpha}})$ and it behaves like a cosmological constant and provides acceleration.

1.5.7 Phantom Dark Energy Models :-

Caldwell [73] pointed out that a very good fit to the luminosity-distance curve (Figure 1.3) can be provided by a dark energy component which violates the weak energy condition so that the equation of state $w < -1$. He dubbed this candidate as *"phantom dark energy"*. A study of high-z supernovae [21] also reveals that the dark energy equation of state has 99% probability of having a value < -1 if no constraints are imposed on Ω_m.

A number of models appeared in the literature where the dynamical nature of phantom energy was constructed by taking a kinetic term with a 'wrong' sign in equation (1.49) so that it can give rise to the present acceleration of the universe [74, 75, 76, 77]. Phantom dark energy models have also been studied in Brans-Dicke theory [78] or in an interacting scenario where the phantom field is coupled to some other field [79, 80]. However, these models suffer from the problem of instability at the quantum level [81] as there is no proper ground state because of the negative kinetic energy. Also models with $w < -1$ suggest that the effective velocity of sound in the medium $v = \sqrt{|dp/d\rho|}$ can become larger than the velocity of light. The phantom models imply a pathological behaviour for the cosmological model at a finite future. If t_{eq} denotes the time when matter density and phantom energy density become equal, then the scale factor of the universe grows as

$$a(t) \approx a(t_{eq}) \left[(1+w)\frac{t}{t_{eq}} - w \right]^{\frac{2}{3(1+w)}}, \quad w < -1 \ . \tag{1.85}$$

Therefore, when $t \rightarrow \left(\frac{w}{w+1}\right)t_{eq}$, $a(t) \rightarrow \infty$, i.e, the scale factor diverges in a finite time. At that epoch, Hubble parameter H also diverges implying that the expansion rate of the universe reaches an infinite value in a finite time. This situation is termed as '*Big Rip*'. Thus the universe dominated by phantom energy culminates to a future curvature singularity (

See [82, 83, 73, 84, 85, 86, 87, 88, 89, 90, 91]). However, some other models have also been investigated where $w < -1$ is attained without considering a negative sign for the kinetic term. These models are called '*Braneworld models*' [92, 15] which has $w_{eff} < -1$ today, but does not run into a 'Big Rip' in finite future.

1.5.8 Braneworld Models :-

Braneworld cosmology suggests that we could be living on a four dimensional '*brane*' which is embedded in a five or higher dimensional '*bulk*'. It is considered that matter fields are confined to the brane whereas gravity is free to propagate throughout the bulk (for a comprehensive discussion, we refer to the lectures by Roy Maartens [94]). In the Randall-Sundrum (RS) [95] scenario, the equation of motion of a scalar field propagating in the brane is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 , \tag{1.86}$$

where

$$H^2 = \frac{8\pi}{3m^2}\rho\left(1 + \frac{\rho}{2\sigma}\right) + \frac{\Lambda_4}{3} + \frac{\varepsilon}{a^4} , \tag{1.87}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) . \tag{1.88}$$

Here, ε is an arbitrary constant and σ is the brane tension which relates the four-dimensional Planck mass (m) and the five-dimensional Planck mass (M) as

$$m = \sqrt{\frac{3}{4\pi}}\left(\frac{M^3}{\sqrt{\sigma}}\right) . \tag{1.89}$$

Also the four-dimensional cosmological constant Λ_4 on the brane and the five-dimensional cosmological constant Λ_b on the bulk are related as

$$\Lambda_4 = \frac{4\pi}{M^3}\left(\Lambda_b + \frac{4\pi}{3M^3}\sigma^2\right) . \tag{1.90}$$

Equation (1.87) contains an additional term $\frac{\rho}{2\sigma}$ because of which the damping experienced by the scalar field as it rolls down the potential dramatically increases so that inflation can be sourced by potentials, such as $V \propto e^{-\lambda\phi}$, $V \propto \phi^{-\alpha}$ etc, which are normally too steep to produce slow-roll. This gives rise to the possibility that both inflation and quintessence may be obtained from the same scalar field. These models are called '*quintessential inflationary models*' (see [96, 97, 98, 99, 100, 101] and references therein). An example of quintessential inflation is shown in Figure (1.6).
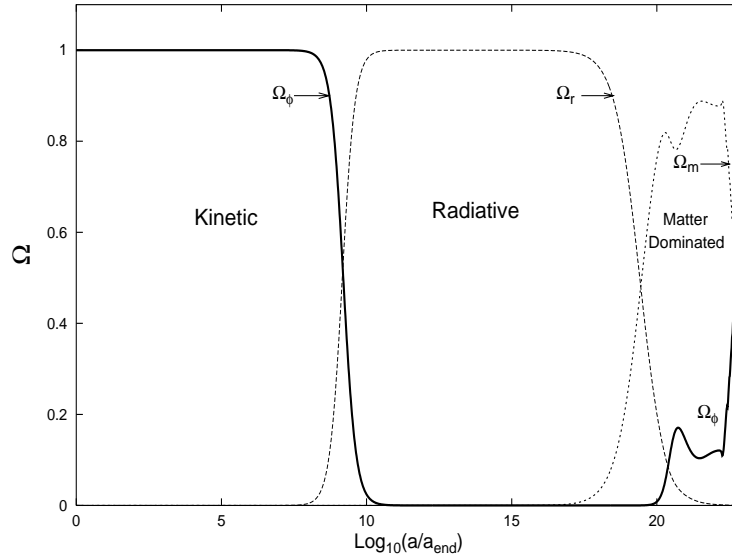
Figure 1.6: *The post-inflationary density parameter Ω is plotted for the scalar field (solid line), radiation (dashed line), and cold dark matter (dotted line) in the quintessential inflationary model. (From Sahni, Sami and Souradeep [99])*

A different way of obtaining an accelerating universe was suggested in the braneworld model developed by Deffayet, Dvali and Gabadadze (DDG) [102, 103]. Here both Λ_b and σ were set to zero, while a curvature term was introduced in the brane action so that it takes

the form

$$S = M^3 \int_{bulk} R + m^2 \int_{brane} R + \int_{brane} L_{matter} . \qquad (1.91)$$

The resulting Hubble parameter in the DDG braneworld model is

$$H = \sqrt{\frac{8\pi G \rho_m}{3} + \frac{1}{l_c^2}} + \frac{1}{l_c} , \qquad (1.92)$$

where $l_c = \frac{m^2}{M^3}$ is a new length scale. An important property of this model is that the acceleration of the universe is not obtained from any '*dark energy*' component. Since gravity becomes five dimensional on length scales $R > l_c = 2H_0^{-1}(1 - \Omega_m)^{-1}$, it is seen that the expansion of the universe is modified during late times instead of early times as in the RS model.

A more general class of braneworld models, which includes RS cosmology and DDG brane is described by the action [104, 105]

$$S = M^3 \int_{bulk} (R - 2\Lambda_b) + \int_{brane} (m^2 R - 2\sigma) + \int_{brane} L_{matter} . \qquad (1.93)$$

For $\sigma = \Lambda_b = 0$, it gives back the DDG model, whereas for $m = 0$ it reduces to RS model. Sahni and Shtanov [92] have shown that the Hubble parameter for this action comes out as

$$\frac{H^2(z)}{H_0^2} = \Omega_m(1 + z)^3 + \Omega_\sigma + 2\Omega_l \mp 2\sqrt{\Omega_l}\sqrt{\Omega_m(1 + z)^3 + \Omega_\sigma + \Omega_l + \Omega_{\Lambda_b}} , \qquad (1.94)$$

where $\Omega_l = \frac{1}{l_c^2 H_0^2}$, $\Omega_m = \frac{\rho_{0m}}{3m^2 H_0^2}$, $\Omega_\sigma = \frac{\sigma}{3m^2 H_0^2}$, $\Omega_{\Lambda_b} = -\frac{\Lambda_b}{6H_0^2}$. (The $\mp$ sign refers to two different ways in which the brane can be embedded in the bulk (for details see [92]).

An important feature of the braneworld model given by equation (1.94) is that it can lead to an effective equation of state of dark energy $w_{eff} \leq -1$. Also it has been shown that in this model, the acceleration of the universe can be a transient phenomenon which ends once the universe returns to matter dominated expansion after the current accelerated phase of expansion and hence does not fall into the problem of 'Big Rip'.

1.6 Outline of the Present Thesis :

Whatever we have discussed so far indicates that our universe at present is undergoing an accelerated expansion preceded by a decelerated one. This means that the deceleration parameter q must have a signature flip from a positive value to a negative one in the recent past during the matter dominated era itself. This indeed leads to the search for a candidate which can drive this transition from deceleration to acceleration. This thesis is a collection of papers which investigates some cosmological solutions where this transition is obtained with or without the need of "*dark energy*".

The investigations carried out will be presented in four chapters following the introduction. The first of them (chapter-2) consists of one paper entitled "*Acceleration of the universe with a simple trigonometric potential*". In this paper we consider a minimally coupled scalar field ϕ, with some potential $V(\phi)$, as the driver of the late time acceleration. The form of deceleration parameter q as a function of scale factor a has been chosen such that it has the desired property of signature flip and then from this the scalar field and required potential are found out.

The flat Robertson-Walker line element is written as

$$ds^2 = dt^2 - a(t)^2[dr^2 + r^2 d\Omega^2] \ . \tag{1.95}$$

The expression for the deceleration parameter q is taken as,

$$q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2} = -1 - \frac{pa^p}{(1 + a^p)} \tag{1.96}$$

where p is a constant. It is found that for $-2 < p < -1$, the deceleration parameter q indeed has the property of a signature flip. Equation (1.96) integrates to yield

$$H = \frac{\dot{a}}{a} = A(1 + a^p), \quad A > 0 \; , \tag{1.97}$$

where A is an arbitrary constant of integration.

In the present work, the problem is completely worked out for $p = -\frac{3}{2}$. The reason for choosing this particular value of p is that it yields $q = 0.5$ for a very low value of a. For a flat matter dominated FRW model without any quintessence matter indeed $q = 0.5$. So the matter dominated universe has a chance to evolve from the standard setting where the formation of structures is facilitated. Furthermore, the transition from the radiation dominated era to the matter dominated one is quite well understood for $q = 0.5$ on the matter-dominated side.

As we are interested in the matter dominated epoch, the fluid is taken in the form of pressureless dust and the Einstein equations are written with a minimally coupled scalar field ϕ having a potential $V(\phi)$.

With the form of the deceleration parameter assumed in equation (1.96), the Einstein field equations are solved and it is found that the potential $V(\phi)$ is obtained as a trigonometric function of the scalar field ϕ, viz, $V(\phi) = \frac{9A^2}{2} \cot^2 \left(\frac{\sqrt{3}\phi}{4} \right) + 3A^2$. Also the expressions for the dimensionless density parameter Ω_m for the normal matter and Ω_ϕ for the quintessence matter are worked out for this model and it is found that the values obtained are well within the constraint range obtained from observations [26, 2, 107].

The equation of state parameters w for the total matter (w_t) and that for the quintessence matter (w_ϕ) are calculated and are found to be consistent with observational requirements.

This investigation shows that we can have a single analytical expression for q which gracefully transits from deceleration to acceleration and thus has a better footing than the piecewise

solutions. Also the form of the potential obtained here is a simple trigonometric function of ϕ and thus adds to the list of quintessence potentials that serve the purpose of providing a presently accelerating universe [108].

The chapter 3 consists of one paper entitled *"Spintessence : A possible candidate as a driver of the late time cosmic acceleration"*. In this paper we work with a complex scalar field model, namely spintessence, proposed by Boyle et al. [109]. The scalar field is taken as,

$$\Psi = \phi_1 + i\phi_2 \; , \tag{1.98}$$

which can also be written as

$$\Psi = \phi e^{i\omega t} \tag{1.99}$$

such that the complex part is taken care of by a phase term. This kind of complex scalar fields are already known in the literature with reference to the '*cosmic strings*' [110]. However, cosmic strings have a very specific form of potential whereas in quintessence models, the form of potential has to be found out.

In the present work it has been shown completely analytically that the deceleration parameter q indeed has a signature flip from positive to negative values which indicates an early deceleration and a present acceleration. In the spintessence model proposed by Boyle et al. [109], it has been considered that ω is a slowly varying function of time which indeed can be approximated to be a constant over the entire period of dust dominated era. In the present work, we also choose ω to be a constant. This choice along with the field equations leads to an equation

$$a^3 \frac{dH}{dt} = -l - mH^2 \; , \tag{1.100}$$

where a is the scale factor of the universe, $H = \frac{\dot{a}}{a}$ is the Hubble parameter and l, m are positive constants, which involve the parameters of the theory and constants of integration.

Now, we make a transformation of time co-ordinate as,

$$\frac{dx}{dt} = \frac{1}{a^3} \tag{1.101}$$

which being positive definite indicates that x is a monotonically increasing function of t. We now express the various parameters of the model as functions of new cosmic time x without any loss of generality or distortion of events.

With these choices, the Hubble parameter H and the deceleration parameter q comes out in terms of the new variable x as,

$$H = n \tan(\beta - mnx) \tag{1.102}$$

and

$$q = -1 + \frac{1}{a^3} \left(\frac{l}{H^2} + m \right) . \tag{1.103}$$

It is evident from equation (1.103) that q has a zero when

$$a_1{}^3 = \frac{l}{H_1^2} + m , \tag{1.104}$$

the suffix '1' indicates the values of the quantities for $q = 0$.

Also from equation(1.103) it has been shown that $\frac{dq}{da}\big|_1$ is negative definite which indicates that q is a decreasing function of a, at least when $q = 0$. Thus, q definitely enters a negative value regime from a positive value at $a = a_1$ and $H = H_1$. So, it has been shown completely analytically that a spintessence model can very well serve the purpose of providing the signature flip in q. The important result achieved here is that if q changes its sign in course of evolution, this change will be in the right direction, i.e, from positive to negative. Also it

has been shown that the flip occurs irrespective of the particular form of potential which is a bonus as the form of the quintessence potential is yet to be specified. Another important feature of this work is that the value of the scale factor a, where this flip in q takes place, can be expressed in terms of parameters of the theory and various constants of integration. The latter are arbitrary constants and thus can be adjusted to fit into the observational results.

The chapter 4 consists of two papers where attempts have been made to explain the late time acceleration of the universe in the framework of scalar tensor theories. In the first paper we work out the problem in Brans-Dicke theory [43] whereas in the second paper we work in Nordtvedt's theory [57] which is a generalized version of Brans-Dicke theory.

The first paper in this chapter is entitled "*A late time acceleration of the universe with two scalar fields : many possibilities*" and addresses the '*graceful entry*' problem of how the universe transits from a decelerating phase of expansion to an accelerated one. The basic motivation for using two scalar fields stems from existing literature on inflation. Mazenko, Wald and Unruh [111] showed that a classical slow roll is invalid if the scalar field driving inflation is self interacting. Also it has been shown that a single scalar field with a slow roll puts generic restrictions on the potentials driving inflation [112]. These problems led to the belief that a successful inflationary model requires two scalar fields [113]. The present work uses this idea of introducing two scalar fields. One of these fields is responsible for the present acceleration of the universe, called the quintessence field, which interacts with the other such that the quintessence field has an oscillatory behaviour at the beginning of the matter dominated epoch and grows later so as to dominate the dynamics of the universe. None of the quintessence fields already there in literature has a proper physical motivation and it will be even more embarrassing to introduce a second field without any underlying physics.

So, the better arena is provided by a scalar tensor theory, such as Brans-Dicke theory, where one scalar field is already there in the purview of the theory and is not put in by hand.

The relevant action in Brans-Dicke theory is given by

$$S = \int \left[\frac{\psi R}{16\pi G_0} - \omega \frac{\Psi_{,\mu}\Psi^{,\mu}}{\Psi} - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - U(\Psi,\phi) + L_m \right] \sqrt{-g}d^4x \;, \qquad (1.105)$$

where Ψ is the Brans-Dicke scalar field, R is the Ricci scalar, G_0 is the Newtonian constant of gravitation, ω is the dimensionless Brans-Dicke parameter, ϕ is the quintessence scalar field and $U(\Psi,\phi)$ is the potential via which the two fields interact amongst themselves. We choose the form of $U(\Psi,\phi)$ as $V(\phi)\Psi^{-\beta}$ which was used by Banerjee and Ram [52] for finding a solution of the graceful exit from inflationary paradigm in Brans-Dicke theory. With a slow roll approximation ($\dot{\phi}^2, \ddot{\phi} \approx 0$), the conditions for an initially oscillating ϕ which grows during later stages are found out for two examples - a power law expansion and an exponential expansion of the scale factor a.

For a power law expansion of the scale factor of the form

$$a = a_0 t^n, \quad n > 1, \qquad (1.106)$$

the condition on the potential $V(\phi)$ comes out as

$$(lnV)''\big|_i \approx -\frac{1}{A}t_i^{3n-2} \qquad (1.107)$$

where A is a constant involving the parameters of the model. The subscript 'i' stands for some initial time ($t = t_i$).

Similarly, for the exponential expansion of the form

$$a = a_0 e^{\alpha t}, \quad a_0, \alpha > 0 \;, \qquad (1.108)$$

the condition on the potential comes out as

$$(\ln V)''\big|_i \approx 0 \ . \tag{1.109}$$

This indicates that for an exponentially expanding universe, the quintessence potential $V(\phi)$ should be exponential at least in the beginning.

In this paper two simple examples have been considered. However, this work can be extended for more complicated kind of accelerated expansion of the universe. A key feature of this work is that here the numerical value of the Brans-Dicke parameter ω is not severely constrained to very low values in order to drive the acceleration. For example, in the exponential expansion case, ω is completely arbitrary whereas in power law expansion case it has been found that value of ω can be determined in terms of other free parameters of the model. So, in both the cases ω can be adjusted to some high value which is compatible with the limits imposed by solar system experiments [114].

In the second paper in this chapter entitled "*An interacting scalar field and the recent cosmic acceleration*", the possibility of transition from deceleration to acceleration has been studied in the framework of a generalised scalar tensor theory.

The dark matter and dark energy components are usually considered to be non-interacting and their evolutions are considered to be independent of each other. Recently Zimdahl and Pavon [115, 116] showed that the interaction between the dark matter and dark energy components could be useful in solving the coincidence problem and an interacting scenario may provide a more general and better framework for obtaining an accelerating universe (see also [117, 118, 119]) .

In this paper, we introduce an interaction between the dark matter and the geometrical scalar field ϕ. The Brans-Dicke field equations are written in so called "Einstein frame"

as discussed earlier. The field equations in this version look tractable, although one has to sacrifice the equivalence principle as the rest mass of a test particle becomes a function of the scalar field ϕ. However, in this work, the metric has been transformed back to the original atomic units and the conclusions are drawn only from this.

In place of the usual matter conservation equation

$$\dot{\bar{\rho}}_m + 3\bar{H}\bar{\rho}_m = 0 \ , \tag{1.110}$$

we have chosen an interaction between the dark matter and the scalar field ϕ of the form

$$\dot{\bar{\rho}}_m + 3\bar{H}\bar{\rho}_m = -\alpha\bar{H}\bar{\rho}_m \tag{1.111}$$

where $\bar{\rho}_m$ is the matter density of the universe, $\bar{H}$ is the Hubble parameter, α is a positive constant. The overhead bars indicate that the equations are written in the conformally transformed version (Einstein's frame) where the transformed metric components are related to the original ones as

$$\bar{g}_{\mu\nu} = \phi g_{\mu\nu} \ . \tag{1.112}$$

The negative sign in equation (1.111) indicates that the energy is transferred from the dark matter component to the scalar field ϕ. It is found that this particular choice of interaction gives rise to a simple power law solution of the scale factor a - which can be either ever accelerating or ever decelerating depending upon the choice of parameters. Obviously we are not interested in such a scenario which does not provide the transition from deceleration to acceleration. One way out of this problem is to consider a generalization of Brans-Dicke theory where the parameter ω is a function of the scalar field ϕ rather than being a constant [57].

We make a choice of ω as

$$\frac{2\omega + 3}{4} = \frac{\alpha}{(3+\alpha)^2} \frac{\phi}{\left(\sqrt{\phi} - 1\right)^2} \ , \tag{1.113}$$

α being a positive constant.

With this choice we obtain the solution for the scale factor in original units as

$$a = \frac{At^{\frac{2}{(3+\alpha)}}}{(1 - \phi_0 t)} \;, \tag{1.114}$$

ϕ_0, A are constants.

From equation (1.114) one can easily find out the expression for q in the original version

as

$$q = -1 + \frac{\frac{2}{3+\alpha}(1 - \phi_0 t)^2 - \phi_0{}^2 t^2}{[\frac{2}{3+\alpha}(1 - \phi_0 t) + \phi_0 t]^2} \;. \tag{1.115}$$

The q vs. t plot shows the transition from positive value to a negative value for small values

of α and also the nature of the curve is not critically sensitive to small changes in the value

of α.

However, from equation (1.114) it is evident that at $t \to \frac{1}{\phi_0}$, a, H and q all blow up. So

the model is not valid upto infinite future and has a future singularity. So the model mimics

a phantom model and has a big rip.

Also, it is evident from equation (1.113) that $(2\omega + 3)$ has to be positive definite to allow a

consistent model. But ω does not have any stringent limit and thus it is possible to adjust ω

to some high value, compatible with the limits imposed by the solar system experiments [114].

Also equations (1.113) and (1.114) reveal that in the $\omega \to \infty$ limit, $a \to t^{2/3}$ for small values

of α. This is consistent with the notion that Brans-Dicke theory yields General Relativity in

the infinite ω limit. Also for this model we have calculated the statefinder parameters $\{r, s\}$

introduced by Sahni et. al [11, 120] and found that the nature of the r vs. s plot for small

values of α is similar to the one expected for scalar field quintessence models.

The salient feature of this model is that no dark energy component is required here. The

interaction between the geometrical scalar field and matter component can drive the present acceleration of the universe. Although a particular form of interaction has been chosen, this is by no means unique and other forms of interaction can also be tried.

The chapter 5 consists of one paper entitled *"Curvature driven acceleration : a utopia or a reality?"*. This work explores the possibility whether geometry itself can serve the purpose of explaining the present accelerated expansion without having to resort to some exotic scalar field models. A substantial amount of work has been done along this direction which can provide an accelerating model for the universe [62, 63, 7, 5, 6, 8, 68, 10] as discussed earlier. However, none of them could provide a smooth transition from deceleration to acceleration demanded by both theory [121, 122] and observations [123].

In the present work, we have written the field equations with a general $f(R)$ and studied the model for two specific cases, namely, $f(R) = R - \frac{\mu^4}{R}$ and $f(R) = e^{-R/6}$. In both the cases it is shown that the required transition from deceleration to acceleration can be obtained. The relevant action for a general $f(R)$ is

$$A = \int \left[\frac{1}{16\pi G} f(R) + L_m \right] \sqrt{-g} d^4 x \ . \tag{1.116}$$

For a spatially flat Robertson-Walker spacetime, where

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \ , \tag{1.117}$$

we write down the field equations for a vacuum universe taking $L_m = 0$. In the first example we make a choice of $f(R)$ as

$$f(R) = R - \frac{\mu^4}{R}, \tag{1.118}$$

where μ is a constant. This is exactly the form used by Carroll [5] and Vollick [6].

For this particular choice, from the field equations we arrive at a relation

$$2\dot{H} = \frac{1}{(R^2 + \mu^4)}\left[2\mu^4\frac{\ddot{R}}{R} - 6\mu^4\frac{\dot{R}^2}{R^2} - 2\mu^4 H\frac{\dot{R}}{R}\right] , \qquad (1.119)$$

where the Ricci scalar R is given by

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) . \qquad (1.120)$$

Equation (1.119) is a highly nonlinear equation involving fourth order derivative of the scale factor a and it is difficult to obtain a complete analytic solution for a. But as we are interested in the evolution of the deceleration parameter q, we translate equation (1.119) into the evolution equation for q by using the relation

$$q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2} = -\frac{\dot{H}}{H^2} - 1 . \qquad (1.121)$$

Also, the time derivatives are replaced by the derivatives w.r.t. H. The resulting equation takes the form

$$\frac{1}{3}q^{\dagger\dagger}(q^2-1)H^2 - \frac{2}{3}H^2 q^{\dagger2}(q+2) - \frac{1}{3}q^\dagger H(q-1)(4q+7) - (2q+1)(q-1)^2 + H^4(q-1)^4 = 0 , \quad (1.122)$$

where a † sign indicates differentiation w.r.t. H.

This equation is also highly nonlinear and cannot be solved analytically. But if one can provide two initial conditions for q and $q^\dagger$, a numerical solution is on cards. So, we pick up sets of values for q and $q^\dagger$ for $H = 1$ (i.e, the present values) from observationally consistent regions [11, 120] and plot q versus H numerically. The plot definitely shows a signature flip in q from positive towards negative. Also, the nature of the curve is not critically sensitive to the initial conditions chosen.

In the second example, the form of $f(R)$ is chosen as

$$f(R) = e^{-R/6} . \qquad (1.123)$$

Following the same method as before, the evolution equation for q is written as a function of H. With similar initial conditions for q and $q^\dagger$ at $H = 1$, q is numerically plotted against H and it is found that the curve has features similar to the previous example, i.e, q has a signature change from positive to negative in the recent past. The added feature of this latter example is that q has another signature flip from negative to positive direction indicating a decelerated expansion again in near future.

So, in the present work both the examples indicate that gravity in its own right can lead to a late surge of accelerated expansion having a past deceleration which is essential for explaining nucleosynthesis and the structure formation of the universe. An added advantage of the second example is that in this case the universe re-enters a decelerated expansion phase in the near future and thus '*phantom menace*' is avoided, i.e, the universe does not attain infinite rate of expansion in a finite future.

There is already some criticism of $\frac{1}{R}$ gravity regarding its stability [11], but still there are reasons to be optimistic about a curvature driven acceleration. It deserves mention that although the $\frac{1}{R}$ term has been quite widely used in the literature, it suffers from the drawback that at $R = 0$, i.e, in the late stage of the evolution, the model has a singularity. The other example, i.e, $e^{-R/6}$ is regular everywhere. The present work deals with a vacuum universe, but in a more general framework one has to either put in matter or should derive the relevant matter from curvature itself. Thus the modified theory of gravity provides a platform for more ambitious and detailed work in future.

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Chapter 2

A Simple Quintessence Field

2.1   Acceleration of the Universe with a Simple Trigonometric Potential

(Journal reference : N. Banerjee and S. Das, Gen. Rel. Grav., 37, 1695 (2005); astro-ph/0505121.)

2.1.1 Introduction

Over the last few years, there are growing evidences in favour of the scenario that the universe at present is expanding with an acceleration. The supernovae project [1] and also the Maxima [2] and Boomerang [3] data on cosmic microwave background (CMB) strongly suggest this acceleration. The very recent WMAP data [4] also seem to confirm this. This result is indeed counter-intuitive, as gravity holds matter together and it might be expected that in the absence of any exotic field, the universe should be decelerating. As this acceleration could be brought about by an effective negative pressure, the first choice of candidate for this 'dark energy' had been the 'cosmological constant' or a time varying cosmological parameter $\Lambda(t)$. Due to well-known reasons, Λ has fallen from grace (for an excellent uptodate review, see [5],[6]). A scalar field with a positive definite potential can indeed give rise to an effective negative pressure if the potential term dominates over the kinetic term. The pressure to density ratio for the scalar field, as required by the supernovae observations, is given as $w_\phi < -\frac{2}{3}$ (See ref [7] and references therein). This source of energy is called the quintessence matter (Q-matter). In this context, non-minimally coupled scalar fields had been investigated thoroughly to check if they could drive an accelerated expansion [8]. Brans-Dicke's scalar field appears to generate sufficient acceleration in the matter era, but it has its problems in the earlier evolution [9]. A viscous fluid along with a Q-matter could also be a useful candidate and this appears to solve the coincidence problem also [10]. This coincidence problem, i.e, why the Q-matter dominates only recently, was tackled in the so called tracker solutions [11] where the scalar field energy density runs parallel to the matter energy density from below through the evolution and gets to dominate only during later stages. Very recently Chaplygin gas, which has a nonlinear contribution of the energy density to the dynamics of the model,

has also been invoked [12]. Most of these models do exhibit an accelerated expansion in the matter-dominated regime.

It deserves mention that the same model should have a deceleration in the early phase of matter era in order to provide a perfect ambience for structure formation. Furthermore, the accelerated phase is perhaps only a very recent one. There are observational evidences too that beyond a certain value of the redshift ($z \sim 1.7$), our universe had been going through a decelerated expansion [13]. This indication is indeed reassuring, as the formation of structure in the universe is better supported by a decelerating model. This is because local inhomogeneities will grow and become stable from the seeds of density fluctuation only if the force field is attractive.

Amendola [14] has argued that all the required structure formation and other relevant observations regarding the supernovae could well be explained even if the alleged acceleration of universe started quite a long time back, even beyond $z = 5$. However, this work also shows that the model requires both an accelerated and a decelerated phase of expansion. But a more recent work by Padmanabhan and Roychowdhury [15] shows a striking result. It indicates that if we take the complete data set, i.e, acceleration upto a certain z and deceleration beyond that (i.e, for higher z), then only this conclusion of the change of signature of the deceleration parameter holds. On the other hand, the individual data sets of the high and low redshift supernovae may well be consistent with a decelerating universe without any 'dark energy'.

So indeed we are in need of some form of fields which governs the dynamics in such a

way that the deceleration parameter becomes negative well into the matter era. One such Q-matter had been given by Sen and Sethi [16] where they include a potential which is a 'double exponential' of the scalar field. They obtained a scale factor which is a sine hyperbolic function of time in the matter dominated regime. The deceleration parameter (q) indeed has a sign flip and with a little fine-tuning, the scale factor can grow during the early stages as $t^{2/3}$ which is indeed the usual solution for the Einstein equations for a flat FRW spacetime for pressureless dust.

In the present work, we adopt the following strategy. We choose a form of q as a function of the scale factor a so that it has the desired property of a signature flip. Then with this input, the scalar field and the required potential are found out. It turns out that a fairly simple trigonometric potential does the needful. The origin of the scalar potential, however, cannot be indicated. Surely this is not the ideal way to find out the dynamics of the universe, as here the dynamics is assumed and then the fields are found out without any reference to the origin of the field. But in the absence of more rigorous ways, this kind of investigations collectively might finally indicate towards the path where one really has to search. This *'reverse'* way of investigations had earlier been used extensively by Ellis and Madsen [17] for finding out the potential driving inflation, i.e, an accelerated phase of the universe at a very early stage of its evolution.

2.1.2   Results

For a spatially flat Robertson-Walker spacetime

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\Omega^2], \tag{2.1}$$

the deceleration parameter q is given by

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{2.2}$$

where a is the scale factor of the universe and is a function of the cosmic time 't' alone.

In order to get a model consistent with observations, one needs an expanding universe, i.e, a positive Hubble parameter $H = \frac{\dot{a}}{a}$ throughout the evolution, but a deceleration parameter q, unlike being a positive constant throughout the matter era at $q = 1/2$ as believed until the recent observations, should be a function of the scale factor (or that of t). Furthermore, this functional dependence should be such that q undergoes a transition from its positive phase to a negative one in the matter dominated period itself. It is thus imperative that the scale factor cannot have a simple power-law behaviour. If $a \sim t^n$, the universe will have an accelerated or a decelerated expansion for $n > 1$ or $n < 1$ respectively throughout the period.

In the quest for a varying q consistent with observations, in the same line as that floated by Ellis and Madsen, we propose the relation

$$q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2} = -1 - \frac{pa^p}{1 + a^p}, \tag{2.3}$$

where p is a constant. It is found that for a certain range of negative values of p, this works remarkably well.

The equation (2.3) integrates to yield

$$H = \frac{\dot{a}}{a} = A(1 + a^p) \tag{2.4}$$

where A is an arbitrary constant of integration. A is taken to be positive, which ensures the positivity of the Hubble parameter (the expansion of the universe is never denied!) irrespective of the signature or value of the constant p.

It is found that for values of p between -2 and -1, the model shows exactly the behaviour which is desired (as shown in Figure 2.1).
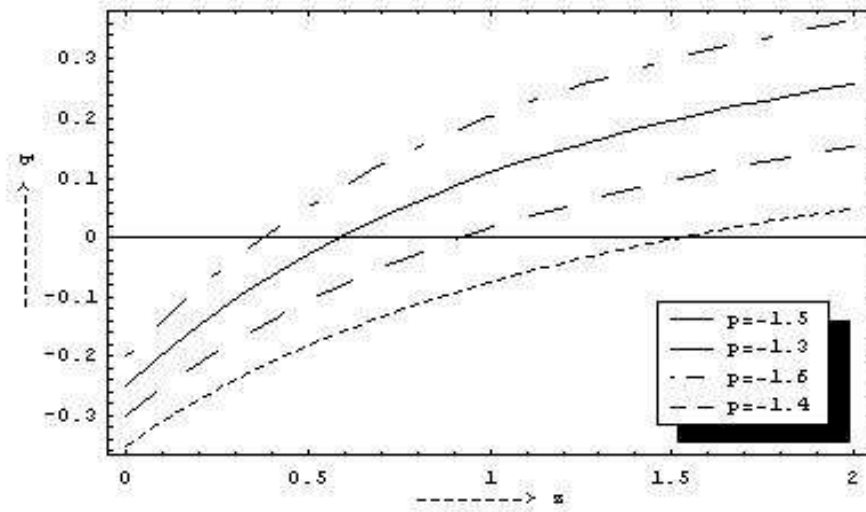
Figure 2.1: *Plot of q vs. z for spatially flat dust dominated R-W model.*

In what follows, we work out the problem completely for $p = -3/2$, for which the model works with a non minimally coupled scalar field with a potential expressed as a simple trigono-metric function of the scalar field.

As the interest is in a matter dominated universe, the fluid is taken in the form of a pressureless dust. The Einstein equations for the space-time given by equation (2.1) are,

$$3\frac{\dot{a}^2}{a^2} = \rho + \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{2.6}$$

where ρ is the density of matter, ϕ is the scalar field and $V(\phi)$ is a scalar potential. The wave equation for the scalar field is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'(\phi) = 0 \tag{2.7}$$

In all equations an overhead dot implies differentiation w.r.t. time and a prime is that w.r.t. the scalar field ϕ. The matter conservation equation, which can in fact be obtained from these three equations in view of the Bianchi identity, yields

$$\rho = \frac{\rho_0}{a^3}, \tag{2.8}$$

ρ_0 being a constant. So we have three equations to solve for four unknowns.

We assume the deceleration parameter as given in equation (2.3), which can be integrated twice to give $H = \dot{a}/a$ as in equation (2.4) and the scale factor as

$$a = [e^{-Apt} - 1]^{-\frac{1}{p}} \tag{2.9}$$

Now, the system of equations (2.5), (2.6) and (2.7) is closed with the assumption of equation (2.3) or equivalently equation (2.4). So in order to solve the system completely, the parameter 'p' should have a fixed value. We choose $p = -\frac{3}{2}$, as it yields $q = 0.5$ for a very low value of $\frac{a}{a_0}$, where a_0 is the present value of the scale factor. The physical motivation for choosing this value of q for early matter dominated epoch is that for a spatially flat FRW model with $p = 0$ without any Q-matter indeed has $q = 0.5$ and that the transition from radiation to matter dominated epoch for this value of q is well studied [18].

With $p = -\frac{3}{2}$, equations (2.5) and (2.6) are used to eliminate $V(\phi)$, and $\dot{\phi}$ can be calculated to be

$$\dot{\phi} = \sqrt{3}Aa^{\frac{-3}{4}} = \frac{\sqrt{3}A}{[e^{\frac{3At}{2}} - 1]^{\frac{1}{2}}} \tag{2.10}$$

The scalar field is found out by integrating equation (2.10) as,

$$\phi = \frac{4}{\sqrt{3}}tan^{-1}(e^{+3At/2} - 1)^{\frac{1}{2}}, \tag{2.11}$$

The potential $V(\phi)$ can also be calculated from equations (2.5) and (2.6) first as a function of time and by the use of equation (2.11) as a function of ϕ as,

$$V(\phi) = \frac{9A^2}{2} cot^2(\frac{\sqrt{3}\phi}{4}) + 3A^2 \tag{2.12}$$

Now one has the complete set of the solutions, $a = a(t)$, $\phi = \phi(t)$, $\rho = \rho(t)$ and $V = V(\phi)$ for $p = -3/2$. The solutions, when plugged in the field equations, namely (2.5), (2.6) and (2.7), satisfy all of them provided

$$\rho_0 = 3A^2. \tag{2.13}$$

From equations (2.5) and (2.6), we note that the contribution from the quintessence field ϕ towards the density and effective pressure are given as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.14}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{2.15}$$

respectively. From these, one can write down the expressions for the dimensionless density parameters $\Omega_m = \frac{\rho_m}{3H^2}$ and $\Omega_\phi = \frac{\rho_\phi}{3H^2}$ respectively for the visible matter and the Q-matter.

With $p = -3/2$, the present model yields

$$\Omega_m = \frac{(1+z)^3}{[1 + (1+z)^{-3/2}]^2} \quad, \tag{2.16}$$

and

$$\Omega_\phi = 1 - \Omega_m \,, \tag{2.17}$$

where z is the redshift parameter given by

$$1 + z = \frac{a_0}{a} \; , \tag{2.18}$$

a_0 being the present value of the scale factor. Ω_{m_0}, the present value of Ω_m, comes out to be

0.25 and $\Omega_{\phi_0} = 0.75$. These values are well within the constraints of $0.2 \leq \Omega_{m_0} \leq 0.8$ [19],[5].

Figure 2.2 shows that Ω_m increases with z, i.e, decreases with the evolution of the universe.

Ω_ϕ starts dominating over Ω_m roughly at $z = 0.8$. At the earlier epoch, i.e, at high z, Ω_ϕ is

very small, allowing a conducive matching onto the radiation era for the perfect ambience for
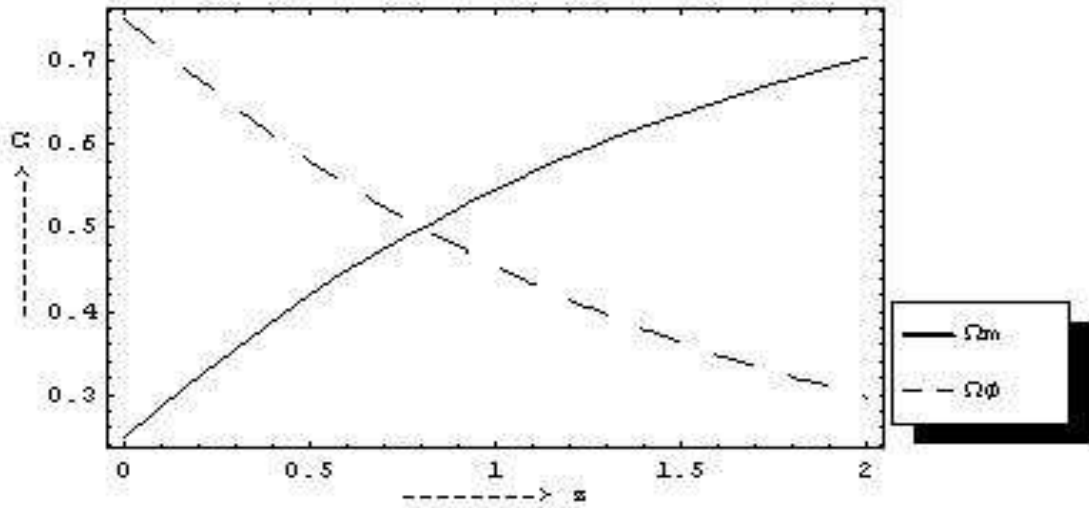
nucleosynthesis.

Figure 2.2: *Plot of Ω vs. z.*

Unlike Newtonian gravity, general relativity ensures that the pressure also contributes in

driving the acceleration of the model. From equations (2.5) and (2.6), one has

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_t + 3p_t) \tag{2.19}$$

where ρ_t and p_t are the total effective density and pressure respectively. If the pressure is connected with the density by the relation

$$p = w\rho, \tag{2.20}$$

the model will accelerate ($\ddot{a} > 0$) only if

$$w_t = \frac{p_m + p_\phi}{\rho_m + \rho_\phi} < -\frac{1}{3}. \tag{2.21}$$

The subscripts 't', 'm' and 'ϕ' stand for total, normal fluid distribution and the Q-matter ϕ respectively. For a matter dominated universe, $p_m = 0$ and hence $w_m = 0$. This particular model gives

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{1 + (1+z)^{\frac{3}{2}}}{-1 - 2(1+z)^{\frac{3}{2}}} \tag{2.22}$$

$$w_t = \frac{p_\phi}{\rho_\phi + \rho_m} = -[1 + (1+z)^{\frac{3}{2}}]^{-1} \tag{2.23}$$

The evolution of w_t versus z and w_ϕ versus z are shown in figure 2.3, which indicates that w_t attains the required value of $-\frac{1}{3}$ or less only close to $z = 0.5$. Beyond that, w_t is less negative, and the universe still decelerates.
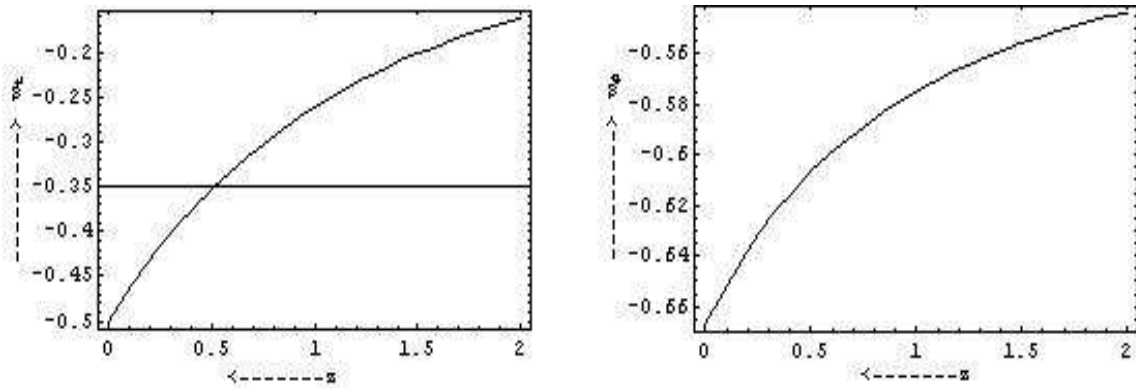

Figure 2.3: *Plot of (a) w_t vs. z and (b) w_ϕ vs. z.*

The value of $w_{\phi 0}$, i.e, the value of the equation of state parameter for the scalar field at

$z = 0$ as given by the present model and as indicated by figure 2.3(b) is definitely within the

constraint range [19].
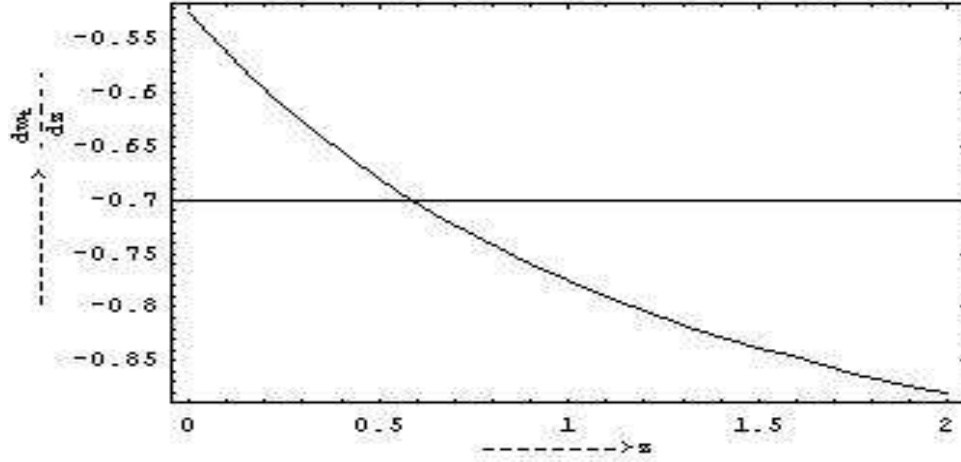
Figure 2.4: *Plot of $\frac{dw_t}{dz}$ vs. z for spatially flat dust dominated R-W model.*

We also plot the rate of change of w_t against z (as shown in figure 2.4). It shows that

$\frac{dw_t}{dz}$ is still negative at the present epoch, but the magnitude of $\frac{dw_t}{dz}$ is decreasing.

The solution for the scale factor is good enough to allow the density contrast to grow

favourably for the formation of large scale structure. Figure 2.5 shows the growth of linearized

density perturbation in this model and evidently indicates that it grows linearly with the scale

factor during later stages as expected for the matter dominated epoch [18].

In the absence of the final form of the quintessence matter, search for the relevant form of

potential will continue and the present investigation is one of them. In view of the high degree

of non-linearity of Einstein's equations, exact solutions always play a vital role as piecewise

solutions have the problem of proper matching at different interfaces. The present model

shows that inspite of the severe constraints imposed by observations, one can still find an

exact FRW model which gives values for the relevant parameters like q, w, Ω_ϕ, Ω_m etc. safely
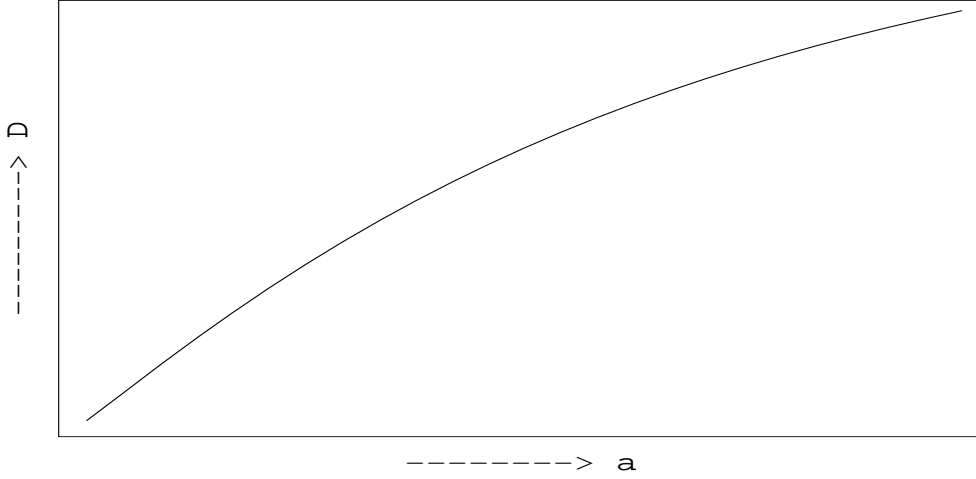
Figure 2.5: *Plot of density contrast D vs. a where $D = \frac{\rho - \bar{\rho}}{\bar{\rho}}$.*

within the range given by observations. It also has the merit of having a single analytical

expression for $q = q(a)$, which gracefully transits from its positive phase to the negative one

and adds to the list of quintessence potentials that serve the purpose of modelling a presently

accelerating universe [20]. Definitely the model has problems, particularly that of fine tuning,

but infact, all quintessence models have some such problems.

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Chapter 3

Complex Scalar Field as Quintessence

3.1   Spintessence : A Possible Candidate as a Driver of the Late Time Cosmic Acceleration

3.1.1 Introduction

The stunning results of the observations on the luminosity - redshift relation of some distant supernovae [1, 2], that the universe is currently undergoing an accelerated phase of expansion, poses a serious challenge for the standard big bang cosmology. As the standard gravitating matter gives rise to an attractive field only, this challenge is negotiated in the standard model by invoking some field which gives rise to an effective negative pressure. For many a reason, a dynamical 'dark energy' is favoured against the apparently obvious choice of a cosmological constant Λ for providing this negative pressure [3]. This dynamical dark energy is called a quintessence matter. It cannot be overlooked that a successful explanation of the formation of structures in the early matter dominated era crucially requires an effectively attractive gravitational field and thus a decelerated phase of expansion must have been witnessed by an early epoch of matter dominated universe itself. Very recently Padmanabhan and Roy Chowdhury [4] showed that the data set, only showing the accelerated phase of expansion, can well be interpreted in terms of a decelerated expansion in disguise. The acceleration only becomes meaningful if the full data set shows deceleration upto a certain age of the universe and an acceleration after that (see also ref [5]). In keeping with such theoretical requirements, actual observations indeed indicate such a shift in the mode of expansion - deceleration upto a higher redshift regime (about $z \sim 1.5$) and acceleration in more recent era, i.e, for lesser values of z [6].

This observation indeed came as a relief, as the formation of galaxies could proceed unhindered in the decelerated expansion phase. It also requires all quintessence models to pass through certain fitness tests, such as the model should exhibit a signature flip of the deceleration parameter q from a positive to a negative value in the matter dominated era itself.

Quite a few quintessence models do exhibit such a signature flip of the deceleration parameter. One very attractive model was that of 'spintessence' proposed by Boyle, Caldwell and Kamionkowski [7]. It essentially works with a complex scalar field and a potential, which is a function of the norm of the scalar field. The scalar field,

$$\psi = \phi_1 + i\phi_2, \tag{3.1}$$

can be written as

$$\psi = \phi e^{i\omega t}, \tag{3.2}$$

i.e, the complex part is taken care of by a phase term. This kind of a scalar field is already known in the literature in describing a 'cosmic string' [8] although it has to be noted that a cosmic string has a very specific form of the potential $V(\phi)$, whereas, the relevant form has to be found out for a quintessence model. Boyle et al discussed the so called spintessence model in two limits separately, namely for a high redshift region and also for a very low redshift region. Apparently this model gives exactly what is required, an acceleration for the low z limit whereas a deceleration for a high redshift limit. In the present investigation, it is shown completely analytically that the model indeed works. For high value of the scale factor a, the deceleration parameter q is negative whereas for a low value of a, q is positive. The most important feature is that the value of a, where the sign flip of q takes place, can be analytically expressed in terms of the parameters of the theory and constants of integration.

3.1.2   Field Equations and Results

If we take the scalar field as given by equation (3.2), Einstein's field equations become

$$3\frac{\dot{a}^2}{a^2} = \rho + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\omega^2\phi^2 + V(\phi) \;, \tag{3.3}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\omega^2\phi^2 + V(\phi), \tag{3.4}$$

where a is the scale factor, ρ is the energy density of matter, V is the scalar potential which is a function of amplitude ϕ of the scalar field, and the phase ω is taken to be a constant. An overhead dot implies differentiation w.r.t. the cosmic time t. In a more general case, ω might have been a function of time.

The matter distribution is taken in the form of dust where the thermodynamic pressure p is equal to zero. This is consistent with the 'matter dominated' epoch. This leads to the first integral of the matter conservation equation as

$$\rho = \frac{\rho_0}{a^3}. \tag{3.5}$$

Variation of the relevant action with respect to ϕ yields the wave equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'(\phi) = \omega^2\phi, \tag{3.6}$$

where a prime is a differentiation w.r.t. ϕ.

As the scalar field actually has two components, a third conservation equation is found as

$$\omega = \frac{A_0}{\phi^2 a^3} \;, \tag{3.7}$$

A_0 being a constant.

In the spintessence model described by Boyle et al [7], it has been considered that ω is a very

slowly varying function of time which indeed can be considered as a constant over the entire

period of dust dominated era. So for constant ω, equation (3.7) yields

$$\phi^2 a^3 = A , \tag{3.8}$$

A being a positive constant.

It deserves mention that only two equations amongst (3.5), (3.6) and (3.8) are independent

as any one of them can be derived from the Einstein field equations with the help of the other

two in view of the Bianchi identities. So, we have four unknowns, namely, a, ρ, ϕ and $V(\phi)$

and four equations, e.g, (3.3), (3.4) and two from (3.5)-(3.8). So this is a determined problem

and an exact solution is on cards without any input. From equations (3.3), (3.4) and (3.5)

one can write

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = -\frac{\rho_0}{2a^3} - \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\omega^2\phi^2 \tag{3.9}$$

which takes the form

$$a^3 \frac{dH}{dt} = -l - mH^2, \tag{3.10}$$

where $H = \frac{\dot{a}}{a}$, the Hubble parameter,

$l = \frac{1}{2}(\rho_0 + \omega^2 A)$ and $m = \frac{9A}{8}$ are positive constants.

In deriving this, the equation (3.8) has been used to eliminate ϕ and $\dot{\phi}$ in terms of a and

$\dot{a}$. Now we make a transformation of time coordinate by the equation

$$\frac{dx}{dt} = \frac{1}{a^3}. \tag{3.11}$$

As $\frac{dx}{dt}$ is positive definite (a is the scale factor and cannot take negative values), we find

that x is a monotonically increasing function of t. So one can use x as the new cosmic time

without any loss of generality and deformation of the description of events.

In terms of x, equation (3.10) can be written as

$$\frac{dH}{dx} = -l - mH^2 \ , \tag{3.12}$$

which can be readily integrated to yield

$$H = n \ tan(\beta - mnx) \ , \tag{3.13}$$

where $n^2 = \frac{l}{m}$, is a positive constant, and β is a constant of integration.

Now we use this H as a function of the new cosmic time variable x and find out the behaviour of the deceleration parameter q, which is defined as

$$q = -\frac{\dot{H}}{H^2} - 1 = -\frac{H^\dagger}{a^3 H^2} - 1 \ , \tag{3.14}$$

where a dagger indicates differentiation w.r.t. x.

Equation (3.12) now yields

$$q = -1 + \frac{1}{a^3} \left(\frac{l}{H^2} + m \right) \ . \tag{3.15}$$

Both m and l are positive constants. So q has a 'zero', when

$$a_1{}^3 = \frac{l}{H_1^2} + m \ . \tag{3.16}$$

The suffix 1 indicates the values of the quantities for $q = 0$. Furthermore, equation (3.15) can be differentiated to yield

$$\frac{dq}{da} \bigg|_1 = -\frac{1}{a_1^4} \left[\frac{l}{H_1^2} + 3m \right] \ , \tag{3.17}$$

at the point $q = 0$, $a = a_1$ and $H = H_1$. In deriving the equation (3.17), equations (3.10) and (3.16) have been used. The last equation clearly shows that q is a decreasing function of

a atleast when $q = 0$. So q definitely enters a negative value regime from a positive value at $a = a_1$ and $H = H_1$.

For the sake of completeness, the solution for the scale factor a can be found out by integrating equation (3.13) as

$$a = \left[\frac{m}{ln\left|\frac{1}{cos(\beta - mnx)}\right|^3} \right]^{1/3}. \tag{3.18}$$

3.1.3   Discussion

So evidently, the spintessence model proposed by Boyle et al [7] passes the 'fitness test', the deceleration parameter enters into a negative value in a "finite past". In view of the high degree of non linearity of Einstein equations, exact analytic solutions are indeed more dependable, and the present investigation provides that in support of a spintessence model. Also, the constants of the theory (such as ω) and the constants of integration (such as l and m) are still free parameters and hence provides the 'comfort zone' for fitting into the observational results. Another feature of this study is that the results obtained are completely independent of the choice of potential $V = V(\phi)$. This feature provides a bonus, as different potentials, used as the quintessence matter, are hardly well-motivated and do not have any proper physical background. Recently quite a few investigations show that a complex scalar field indeed serves the purpose of driving a late time acceleration [9]. But most of these investigations either invoke the solution in some limit (such as for a large a) or use some tuning of the form of the potential. The present investigation provides a better footing for them as it shows the transition of q analytically. It also deserves mention that Bento, Bertolami and Sen [10] showed the effectiveness of a Chaplygin gas as a quintessence matter. It is interesting to note that under some assumptions this kind of a fluid formally resembles a complex scalar field as

discussed in this work.

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Chapter 4

Acceleration of the Universe in Scalar

- Tensor Theories

4.1 A Late Time Acceleration of the Universe with Two Scalar Fields : Many Possibilities

(Journal reference : N. Banerjee and S. Das, Mod. Phys. Lett. A, 21, 2663 (2006); gr-qc/0605110.)

4.1.1 Introduction

The present cosmic acceleration is now generally believed to be a certainty rather than a speculation. The recent data on supernovae of type Ia suggested this possibility quite strongly [1, 2, 3, 4] and the most trusted cosmological observations, namely that on Cosmic Microwave Background Radiation [5, 6, 7, 8, 9], appear to be quite compatible with an accelerated expansion of the present universe. The natural outcome of these observations is indeed a vigorous search for the form of matter which can give rise to such an expansion, as a normal matter distribution gives rise to an attractive gravity leading to a decelerated expansion. This particular form of matter, now popularly referred to as "dark energy", is shown to account for as much as 70% of the present energy of the universe. This is also confirmed by the highly accurate Wilkinson Microwave Anisotropy Probe (WMAP) [10, 11, 12, 13, 14]. A large number of possible candidates suitable as a dark energy component have appeared in the literature. Excellent reviews on this topic are available [15, 16, 17]. Most of the dark energy candidates are constructed so as to generate an effective pressure which is sufficiently negative driving an accelerated expansion. The alleged acceleration can only be a very recent phenomenon and must have set in during the late stages of the matter dominated expansion of the universe. This requirement is crucial for the successful nucleosynthesis in the radiation dominated era as well as for a perfect ambience for the formation of structure during the matter dominated era. Fortunately, the observational evidences are also strongly in favour of a scenario in which the expansion of the universe had been decelerated (the deceleration parameter $q > 0$) for high redshifts and becomes accelerated ($q < 0$) for low values of the redshift z [18]. So the dark energy sector should have evolved in such a way that the consequent negative pressure has begun dictating terms only during a recent past.

This so easily reminds one about the inflationary universe models where an early accelerated expansion was invoked so as to wash away the horizon, fine tuning and some other associated problems of the standard big bang cosmology. The legendary problem, that the inflationary models themselves had, was that of a "graceful exit" - how the accelerated expansion gives way to the more sedate ($q < 0$) expansion so that the universe could look like as we see it now. For a very comprehensive review, we refer to Coles and Lucchin [19] or Kolb and Turner [20]. The "graceful exit" problem actually stems from the fact that for the potentials driving inflation, the phase transition to the true vacuum is never complete in a sizeable part of the actual volume of the universe. The attempts to get out of this problem involved the introduction of a scalar field which slowly rolls down its potential so that there is sufficient time available for the transition of phase throughout the actual volume of the universe. The current accelerated expansion thus poses the problem somewhat complementary to the graceful exit - that of a "graceful entry".

The present work addresses this problem, and perhaps provides some clue regarding the solution of the problem. The basic motivation stems from existing literature on inflation. Mazenko, Wald and Unruh [21] showed that a classical slow roll is in fact invalid when the single scalar field driving inflation is self interacting. It was also shown that a slow roll with a single scalar field puts generic restrictions on the potentials driving inflation [22]. This kind of problems led to the belief that for a successful inflationary model, one needs to have two scalar fields [23]. The present work uses this idea of utilising two scalar fields, one of them being responsible for the present acceleration of the universe and is called the quintessence field. The second one interacts nonminimally with the former so that the quintessence field has an oscillatory behaviour at the early matter dominated epoch but indeed grows later to dominate

the dynamics of the more recent stages of the evolution. If such a behaviour is achieved, some clue towards the resolution of the graceful entry problem or the coincidence problem may be obtained. There are quite a few quintessence potentials already in the literature [15] which drives a late time acceleration, but none of them really has an underlying physics explaining their genesis. As one is already hard pressed to find a proper physical background of the quintessence field, it will be even more embarassing to choose a second field without any physical motivation. Naturally, the best arena is provided by a scalar tensor theory, such as Brans-Dicke theory, where one scalar field is already there in the purview of the theory and is not put in by hand. It deserves mention that the Brans-Dicke scalar field was effectively used in "extended inflation" in order to get a sufficient slow roll of the scalar field [24, 25]. Later Brans-Dicke theory was used for finding a solution of the graceful exit problem with a large number of potentials [26], where the inflaton field evolves to an oscillatory phase during later stages.

In the next section we write down the field equations in Brans-Dicke (B-D) theory with a quintessence field ϕ, the potential $V(\phi)$ driving acceleration being modulated by the B-D scalar field ψ as $V(\phi)\psi^{-\beta}$. With a slow roll approximation, the conditions for an initially oscillating ϕ, which grows only during later stages, are found out for two examples, a power law expansion and an exponential expansion of the scale factor. The particular form of $V(\phi)$ is quite irrelevant in this context, the conditions only put some restrictions on the constants of the theory and the parameters of the model. So the form of the potential is arbitrary to start with, only the conditions on the parameters and the 'value' of $V(\phi)$ has to be satisfied and hence many possibilities are opened up to accommodate a physically viable potential as the driver of the late time acceleration. However, in some cases, this could restrict the form

of $V(\phi)$ as well. In the last section, we make some remarks on the results obtained.

4.1.2   A Model with a Graceful Entry

The relevant action in Brans-Dicke theory is given by

$$S = \int [\frac{\psi R}{16\pi G_0} - \omega \frac{\psi_{,\mu}\psi^{,\mu}}{\psi} - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - U(\psi,\phi) + L_m]\sqrt{-g}d^4x \tag{4.1}$$

where G_0 is the Newtonian constant of gravitation, ω is the dimensionless Brans-Dicke param-eter, R is the Ricci scalar, ψ and ϕ are the Brans-Dicke scalar field and quintessence scalar fields respectively. If we now choose $U(\psi,\phi)$ as $V(\phi)\psi^{-\beta}$ as explained, the field equations, in units where $8\pi G_0 = 1$, can be written as

$$3H^2 + 3H\frac{\dot{\psi}}{\psi} - \frac{\omega}{2}\frac{\dot{\psi}^2}{\psi^2} = V(\phi)\psi^{-(\beta+1)} + \frac{\rho}{\psi} , \tag{4.2}$$

$$3H\dot{\phi} + V'(\phi)\psi^{-\beta} = 0 , \tag{4.3}$$

$$(2\omega + 3)(\ddot{\psi} + 3H\dot{\psi}) = (\beta - 4)V(\phi)\psi^{-\beta} + \rho , \tag{4.4}$$

where a dot represents differentiation with respect to time t and a prime represents differenti-ation with respect to the scalar field ϕ. As we require the potential $U(\psi,\phi)$ to grow with time so that the effective negative pressure dominates at a later stage, β should be negative for a ψ growing with time or positive for a ψ decaying with time. The field equations are written in the slow roll approximation, i.e, where $\dot{\phi}^2$ and $\ddot{\phi}$ are neglected in comparison to others. $H = \frac{\dot{a}}{a}$ is obviously the Hubble parameter. As we dropped the field equation containing stresses, we can use the matter conservation equation as the fourth independent equation which yields on integration

$$\rho = \frac{\rho_0}{a^3} , \tag{4.5}$$

ρ_0 being a constant. This is so as the fluid pressure is taken to be zero as we are interested in the matter dominated era.

Using the expression for $V(\phi)\psi^{-\beta}$ from equation (4.2) in equation (4.4), we can write

$$(2\omega + 3)[\frac{\ddot{\psi}}{\psi} + 3H\frac{\dot{\psi}}{\psi}] = (\beta - 4)[3H^2 + 3H\frac{\dot{\psi}}{\psi} - \frac{\omega}{2}\frac{\dot{\psi}^2}{\psi^2}] - (\beta - 5)\frac{\rho_0}{a^3\psi} \ . \tag{4.6}$$

If the scale factor a is known, this equation can be integrated to yield the Brans-Dicke field ψ.

Equations (4.3) and (4.4) yield

$$\frac{V(\phi)}{V'(\phi)} = -\frac{(2\omega + 3)(\ddot{\psi} + 3H\dot{\psi}) - \rho}{3(\beta - 4)H\dot{\phi}} \ . \tag{4.7}$$

Hence, if we define

$$f(\phi) = \int \frac{V}{V'}d\phi \ , \tag{4.8}$$

then

$$f(\phi_0) - f(\phi_i) = \int_{t_i}^{t_0} F(t)dt \ , \tag{4.9}$$

where

$$F(t) = -\frac{(2\omega + 3)(\ddot{\psi} + 3H\dot{\psi}) - \rho}{3(\beta - 4)H} \tag{4.10}$$

and subscripts 'o' and 'i' stands for the present value and some initial value, such as the onset of the matter dominated phase of evolution.

For a given $a = a(t)$, therefore, equation (4.6) can be used to find ψ, which in turn, with equations (4.8) and (4.9) determines $f(\phi)$. Now, these equations can be used to put bounds on the values of derivatives of the potential, which would ensure that the dark energy has an oscillating phase in the early stages.

The complete wave equation for the quintessence field ϕ is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi)\psi^{-\beta} = 0.$$

The condition for a small oscillation of ϕ about a mean value is $V'(\phi) = 0$. This provides a kind of plateau for the potential which hardly grows with evolution and hence the dynamics of the universe is practically governed by the B-D field ψ and the matter density ρ. We find the condition for such an oscillation of ϕ at some initial epoch by choosing $V'(\phi) \approx 0$ which yields

$$\left.\frac{\ddot{\phi}}{3H\dot{\phi}}\right|_i \approx -1 \ .$$

During later stages, the universe evolves according to the equations (4.2) - (4.4) where $V'(\phi) \neq 0$, and the scalar field slowly rolls along the potential so that the quintessence field takes an active role in the dynamics and gives an accelerated expansion of the universe.

Two examples, one for power law and the other exponential expansion, will be discussed in the present work.

I. Power law expansion :-

If $a = a_0 t^n$ where $n > 1$, the universe expands with a steady acceleration, i.e, with a constant negative deceleration parameter $q = -\frac{(n-1)}{n}$.

With this,

$$H = \frac{\dot{a}}{a} = \frac{n}{t} \ , \tag{4.11}$$

and equation (4.6) has the form

$$c_1\frac{\ddot{\psi}}{\psi} + c_2\frac{\dot{\psi}}{\psi}\frac{1}{t} + c_3\frac{\dot{\psi}^2}{\psi^2} + c_4\frac{1}{t^2} + c_5\frac{1}{t^{3n}\psi} = 0 \ , \tag{4.12}$$

c_i's being constants given by,

$$c_1 = (2\omega+3), \quad c_2 = 3n(2\omega-\beta+7), \quad c_3 = \frac{\omega}{2}(\beta-4), \quad c_4 = -3n^2(\beta-4), \quad c_5 = (\beta-5)\frac{\rho_0}{a_0^3} \ . \tag{4.13}$$

The simplest solution for ψ in equation (4.12) is

$$\psi = \psi_0 t^{2-3n} \; , \tag{4.14}$$

ψ_0 being a constant.

The consistency condition for this is,

$$(2\omega+3)(2-3n)(1-3n)+3n(2\omega-\beta+7)(2-3n)+\frac{\omega}{2}(\beta-4)(2-3n)^2-3n^2(\beta-4)+(\beta-5)\frac{\rho_0}{a_0^3\psi_0} = 0 \; .$$

From equations (4.9) and (4.10),

$$f(\phi_0) - f(\phi_i) = D \; (t_i^{2-3n} - t_0^{2-3n}) \; , \tag{4.15}$$

where D is a constant involving c_i's, i.e, n, ω, ψ_0, ρ_0 etc. given by

$$D = \frac{(2\omega + 3)\psi_0(2 - 3n) - \frac{\rho_0}{a_0^3}}{3n(\beta - 4)(2 - 3n)} \; .$$

From the form of potential $V = V(\phi)$, $f(\phi)$ can be found from the relation (4.8). So, the different constants will be related by equation (4.15).

If the quintessence field oscillates with small amplitude about the equilibrium at the beginning, i.e, at $t = t_i$ and grows during the later stages of evolution, then $\left.\frac{\ddot{\phi}}{3H\dot{\phi}}\right|_i \approx 1$, which puts more constraints amongst different parameters of the theory.

Equations (4.7) and (4.14) yield

$$\dot{\phi} = A \; t^{(1-3n)} \; (ln \; V)' \; ,$$

where A is a constant given by

$$A = -\frac{(2\omega + 3)\psi_0(2 - 3n) - \frac{\rho_0}{a_0^3}}{3n(\beta - 4)} = -D(2 - 3n) \; .$$

So,

$$\frac{\ddot{\phi}}{3H\dot{\phi}} = -1 + \frac{1}{3n} + \frac{A}{3n} t^{2-3n}(ln\ V)'' \ .$$

The condition for the small oscillation of ϕ close to $t = t_i$ is

$$\left.\frac{\ddot{\phi}}{3H\dot{\phi}}\right|_i \approx -1 \ ,$$

which gives the condition on $(ln\ V)''$ as

$$(ln\ V)''|_i \approx -\frac{1}{A} t_i^{3n-2}. \tag{4.16}$$

II. Exponential expansion :-

Similarly for an exponential expansion at the present epoch, the constraints can be derived.

For such an expansion,

$$a = a_0\ e^{\alpha t} \ ,$$

where a_0, α are all positive constants.

Then,

$$H = \frac{\dot{a}}{a} = \alpha \ \ and \ \ \rho = \frac{\rho_0}{a_0^3}e^{-3\alpha t} \ .$$

Equation (4.6) with this solution has the form

$$b_1\frac{\ddot{\psi}}{\psi} + b_2\frac{\dot{\psi}}{\psi} + b_3\frac{\dot{\psi}^2}{\psi^2} + b_4 + b_5\frac{1}{e^{3\alpha t}\psi} = 0 \ , \tag{4.17}$$

where b_i's are constants given by

$$b_1 = (2\omega+3), \ \ b_2 = 3\alpha(2\omega-\beta+7), \ \ b_3 = \frac{\omega}{2}(\beta-4), \ \ b_4 = -3\alpha^2(\beta-4), \ \ b_5 = (\beta-5)\frac{\rho_0}{a_0^3}. \tag{4.18}$$

A simple solution for ψ is

$$\psi = \psi_0 e^{-3\alpha t} \; , \tag{4.19}$$

ψ_0 being a constant. The consistency condition for this is,

$$9\alpha^2(\beta + \frac{\omega\beta}{2} - 2\omega - 4) - 3\alpha^2(\beta - 4) + (\beta - 5)\frac{\rho_0}{a_0^3\psi_0} = 0 \; . \tag{4.20}$$

Equations (4.9) and (4.10) will put restrictions on the parameters of potential by

$$f(\phi_0) - f(\phi_i) = \frac{\rho_0}{9\alpha^2 a_0^3(\beta - 4)} \left[e^{-3\alpha t_i} - e^{-3\alpha t_0} \right] \; . \tag{4.21}$$

The condition $\frac{\ddot{\phi}}{3H\dot{\phi}} \approx -1$ for the small oscillation of the scalar field at $t = t_i$, with the help of equations (4.7) and (4.19), leads to the interesting result

$$(ln\ V)''\big|_i \approx 0 \; , \tag{4.22}$$

which indicates that for an exponentially expanding present stage of evolution, the quintessence potential behaves exponentially at least at the beginning.

Thus, if $V = V(\phi)$ is given, then equations (4.15) and (4.21) will help finding out the bounds on the values of the scalar field ϕ and the constants appearing in $V(\phi)$ in terms of the initial and final epochs.

The conditions (4.15) or (4.21) put bounds on the potential so that the quintessence field ϕ has an oscillatory behaviour in the beginning. As the solutions in the examples considered has accelerated expansion, the Q-field ϕ has a steady growth at later stages. For a power law expansion, the growth has some arbitrariness as $V(\phi)$ is not specified. For the exponential expansion, however, the growth of ϕ is governed by an exponential potential.

4.1.3 Conclusion

It deserves mention that non-minimally coupled scalar fields were utilised by Salopek et al[27] and Spokoiny [28, 29] in the context of an early inflation, where the field had an oscillation or a plateau at the end of the inflation. In the present case, we need just the reverse for the Q-field, and that is aided by the non-minimally coupled field ψ.

We see that for a wide range of choice of $V(\phi)$, a power law acceleration is on cards, only the value of $V(\phi)$ at some initial stage is restricted by equation (4.16). For an exponential expansion of the scale factor, however, the potential $V(\phi)$ has to be an exponential function of ϕ (in view of equation (4.22)). This investigation can be extended for more complicated kinds of accelerated expansion.

It is true that general relativity is by far the best theory of gravity and the present calculations are worked out in Brans-Dicke (B-D) theory, but this should give some idea about how a second scalar field may be conveniently used to get some desired results. Although B-D theory lost a part of its appeal as the most natural generalization of general relativity (GR) as the merger of B-D theory with GR for large ω limit is shown to be somewhat restricted [30], it still provides useful limits to the solution of the cosmological problems [24]. Another feature of the present work is that the numerical value of ω required is not much restricted. For power law expansion, ω is restricted by equation (4.16) which clearly shows that it can be adjusted by properly choosing values of some other quantities, whereas for exponential expansion, equation (4.22) shows that ω is arbitrary. This is encouraging as it might be possible to get an acceleration even with a high value of ω, compatible with local astronomical observations [31]. B-D theory had been shown to generate acceleration by itself [32], although it had problems with early universe dynamics. B-D theory with quintessence or some modifications

of the theory [33, 34, 35, 36, 37] were shown to explain the present cosmic acceleration, but all these models, unlike the present work, required a very low value of ω, contrary to the local observations.

4.2 An Interacting Scalar Field and the Recent Cosmic Acceleration

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4.2.1 Introduction

Over the last few years, the speculation that our universe is undergoing an accelerated expansion has turned into a conviction. The recent observations regarding the luminosity - redshift relation of type Ia supernovae [1, 2, 3, 4] and also the observations on Cosmic Microwave Background Radiation (CMBR) [5, 6, 7, 8, 9] very strongly indicate this acceleration. These observations naturally lead to the search for some kind of matter field which would generate sufficient negative pressure to drive the present acceleration. Furthermore, observations reveal that this unknown form of matter, popularly referred to as the "dark energy", accounts for almost 70% of the present energy of the universe. This is confirmed by the very recent Wilkinson Microwave Anisotropy Probe (WMAP) data [10, 11, 12, 13, 14]. A large number of possible candidates for this "dark energy" component has already been proposed and their behaviour have been studied extensively. There are excellent reviews on this topic [16, 38].

It deserves mention that this alleged acceleration should only be a very recent phenomenon and the universe must have undergone a deceleration (deceleration parameter $q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2} > 0$) in the early phase of matter dominated era. This is crucial for the successful nucleosynthesis as well as for the structure formation of the universe. There are observational evidences too that beyond a certain value of the redshift z ($z \sim 1.5$), the universe surely had a decelerated phase of expansion [18]. So, the dark energy component should have evolved in such a way that its effect on the dynamics of the universe is dominant only during later stages of the matter dominated epoch. A recent work by Padmanabhan and Roy Choudhury [39, 40] shows that in view of the error bars in the observations, this signature flip in q is essential for the conclusion that the present universe is accelerating.

So, we are very much in need of some form of a field as the candidate for dark energy, which

should govern the dynamics of the universe in such a way that the deceleration parameter q was positive in the early phases of the matter dominated era and becomes negative during the later stages of evolution. One of the favoured choices for the "dark energy" component is a scalar field called a quintessence field (Q-field) which slowly rolls down its potential such that the potential term dominates over the kinetic term and thus generates sufficient negative pressure for driving the acceleration. A large number of quintessence potentials have appeared in the literature and their behaviour have been studied extensively (for a comprehensive review, see [15]). However, most of the quintessence potentials do not have a proper physical background explaining their genesis. In the absence of a proper theoretical plea for introducing a particular Q-field, non-minimally coupled scalar field theories become attractive for carrying out the possible role of the driver of the late time acceleration. The reason is simple; the required scalar field is already there in the purview of the theory and does not need to be put in by hand. Brans - Dicke theory is arguably the most natural choice as the scalar - tensor generalization of general relativity (GR) because of its simplicity and a possible reduction to GR in some limit. Obviously Brans - Dicke (BD) theory or its modifications have already found some attention as a driver of the present cosmic acceleration [33, 34, 35, 36, 37, 41] (see also [42, 43, 44]). It had also been shown that BD theory can potentially generate sufficient acceleration in the matter dominated era even without any help from an exotic Q - field [32]. But this has problems with the required 'transition' from a decelerated to an accelerated phase. Amongst other nonminimally coupled theories, a dilatonic scalar field had also been considered as the driver of the present acceleration [45].

In most of the models the dark energy and dark matter components are considered to be non-interacting and are allowed to evolve independently. However, as the nature of these

components are not completely known, the interaction between them will indeed provide a more general framework to work in. Recently, Zimdahl and Pavon [46, 47] have shown that the interaction between dark energy and dark matter can be very useful in solving the coincidence problem (see also ref [48, 49, 50]). Following this idea, we consider an interaction or 'transfer of energy' between the Brans - Dicke scalar field which is a geometrical field and the dark matter. The idea of using a 'transfer' of energy between matter and the nonminimally coupled field had been used earlier by Amendola [51, 52]. We do it specifically for a modified Brans - Dicke theory. The motivation for introducing this modification of Brans - Dicke theory is the following. In the presence of matter and a quintessence field, with or without an interaction between them, the evolution of net equation of state parameter w plays a crucial role in driving a late surge of accelerated expansion. But WMAP survey indicates that the time variation of w may be very severely restricted [53]. If the late acceleration is driven by an exchange of energy between matter and a geometrical field ϕ, the question of the variation of w would not arise.

We write down the Brans - Dicke field equations in the so called Einstein frame. The field equations in this version look simpler and G becomes a constant. But one has to sacrifice the equivalence principle as the rest mass of a test particle becomes a function of the scalar field [54]. So, the geodesic equation is no longer valid and the different physical quantities loose their significance. Nevertheless, the equations in this version of the theory enables us to identify the energy contributions from different components of matter. However, for final conclusions we go back to the original atomic units where we can talk about the features with confidence. We choose a particular form of the interaction and show that a constant BD parameter ω can not give us the required flip from a positive to a negative signature of

q in the matter dominated era. We attempt to sort out this problem using a modified form of BD theory where ω is a function of the scalar field ϕ [55]. It has been pointed out by Bartolo and Pietroni [56] that a varying ω theory can indeed explain the late time behaviour of the universe. By choosing a particular functional form of ω, we show that in the interacting scenario, one can obtain a scale factor 'a' in the original version (i.e, in atomic units) of the theory so that the deceleration parameter q has the desired property of a signature flip without having to invoke any quintessence field in the model. We also calculate the statefinder pair {r,s}, recently introduced by Sahni et al [57, 58], for this model. The statefinder probes the expansion dynamics of the universe in terms of higher derivatives of the scale factor, i.e, $\ddot{a}$ and $\dddot{a}$. These statefinder parameters along with the SNAP data can provide an excellent diagnostic for describing the properties of dark energy component in future.

4.2.2   Field Equations and Solutions

The field equations for a spatially flat Robertson - Walker spacetime in Brans - Dicke theory are

$$3\frac{\dot{a}^2}{a^2} = \frac{\rho_m}{\phi} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} \ , \tag{4.23}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - 2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} \ . \tag{4.24}$$

The field equations have been written with the assumption that at the present epoch the universe is filled with pressureless dust, i.e, $p_m = 0$. Here ρ_m is the matter density of the universe, ϕ is the Brans - Dicke scalar field, a is the scale factor of the universe and ω is the BD parameter. An overhead dot represents a differentiation with respect to time t.

The usual matter conservation equation has the form

$$\dot{\rho_m} + 3H\rho_m = 0 \quad .$$

But here we consider an interaction between dark matter and the geometrical scalar field and write down the matter conservation equation in the form

$$\dot{\rho_m} + 3H\rho_m = Q \quad , \tag{4.25}$$

such that the matter field grows or decays at the expense of the BD field. The matter itself is not conserved here and the nature of interaction is determined by the functional form of Q. We do not use the wave equation for the BD field here because if we treat equations (4.23), (4.24) and (4.25) as independent equations, then the wave equation comes out automatically as a consequence of the Bianchi identity. It deserves mention that the wave equation will be modified to contain Q which will determine the rate of pumping energy from the BD field to matter or vice-versa. This interaction term Q is indeed a modification of Brans - Dicke theory. But this interaction does not demand any nonminimal coupling between matter and the scalar field ϕ and hence does not infringe the geodesic equation in anyway. In this interaction, the rest mass of a test particle is not modified but rather a "creation" of matter at the expense of the scalar field ϕ (or the reverse) takes place. In a sense, it has some similarity with the "C - field" of the steady state theory [59].

In the Brans - Dicke theory, the effective gravitational constant is given by $G = \frac{G_0}{\phi}$, which is indeed not a constant. Now, we effect a conformal transformation

$$\bar{g}_{\mu\nu} = \phi g_{\mu\nu} \quad .$$

In the transformed version G becomes a constant. However, this transformation has some limitations which have been mentioned earlier. But the resulting field equations look more

tractable. Equations (4.23) and (4.24) in the new frame look like

$$3\frac{\dot{\bar{a}}^2}{\bar{a}^2} = \bar{\rho} + \frac{(2\omega + 3)}{4}\dot{\psi}^2 \ , \tag{4.26}$$

$$2\frac{\ddot{\bar{a}}}{\bar{a}} + \frac{\dot{\bar{a}}^2}{\bar{a}^2} = -\frac{(2\omega + 3)}{4}\dot{\psi}^2 \ , \tag{4.27}$$

and the matter conservation equation takes the form

$$\dot{\bar{\rho}}_m + 3\,\frac{\dot{\bar{a}}}{\bar{a}}\,\bar{\rho}_m = \bar{Q} \ , \tag{4.28}$$

where an overbar represents quantities in new frame and $\psi = ln\phi$. The scale factor and the

matter density in the present version are related to those in the original version as

$$\bar{a}^2 = \phi a^2 \quad and \quad \rho_m = \phi^2\,\bar{\rho}_m \ . \tag{4.29}$$

Now, we choose the interaction $\bar{Q}$ of the form

$$\bar{Q} = -\alpha\,\bar{H}\bar{\rho}_m \ , \tag{4.30}$$

where α is a positive constant. This negative $\bar{Q}$ indicates a transfer of energy from the dark-

matter (DM) component to the geometrical field ϕ.

Equation (4.28) can be easily integrated with the help of equation (4.30) to yield

$$\bar{\rho}_m = \rho_0\,\bar{a}^{(-\alpha-3)} \ , \tag{4.31}$$

where ρ_0 is a constant of integration.

Then, equations (4.26) and (4.27) alongwith equation (4.31) has a solution

$$\bar{a} = A\bar{t}^{2/(3+\alpha)} \tag{4.32}$$

where A is a constant given by

$$A = [\sqrt{\frac{\rho_0}{3-\alpha}}(\frac{3+\alpha}{2})]^{\frac{2}{3+\alpha}} \ .$$

Some arbitrary constants of integration have been put equal to zero while arriving at equation (4.32) for the sake of simplicity.

Using equations (4.27) and (4.32), one can easily arrive at the relation

$$(\frac{2\omega + 3}{4})\dot{\psi}^2 = \frac{4\alpha}{(3+\alpha)^2}\frac{1}{\bar{t}^2} \ . \tag{4.33}$$

If we consider a non-varying ω, equation (4.33) will give rise to a simple power law evolution of ϕ. From equations (4.29) and (4.32), the scale factor a in atomic units will also have a power law evolution - an ever accelerating or an ever decelerating model contrary to our requirement. This is consistent with the exhaustive solutions in Brans - Dicke cosmology obtained by Gurevich et al. [60], where the dust solutions are all power law. This indicates that the chioce of constants of integration in equation (4.32) does not generically change the model. One way out of this problem is to consider a generalization of Brans - Dicke theory where the parameter ω is a function of the scalar field ϕ rather than a constant [55]. An evolving ω will be a contributory factor in determining the dynamics of the universe.

We make a choice of ω as,

$$\frac{2\omega + 3}{4} = \frac{\alpha}{(3+\alpha)^2}\frac{\phi}{(\sqrt{\phi} - 1)^2} \ . \tag{4.34}$$

Then, equation (4.33) can be integrated to yield

$$\phi = (1 - \phi_0 \bar{t})^2 \ , \tag{4.35}$$

ϕ_0 being a positive constant.

It deserves mention here that since g_{00} and $\bar{g}_{00}$ are both equal to one, the time variable transforms as

$$d\bar{t}^2 = \phi dt^2 \ .$$

This along with equation (4.35) gives

$$\bar{t} = \frac{1}{\phi_0} \left[1 - \sqrt{2\phi_0(t_0 - t)} \right] \ , \tag{4.36}$$

which is a monotonically increasing function of t until $t = t_0$, beyond which the model really does not work. So one can use $\bar{t}$ itself as the new cosmic time in the original version of the theory without any loss of generality. So, for the sake of convenience, from now onwards, we write t in place of $\bar{t}$.

We transform the scale factor back to the original units by equation (4.29), so that we are armed with the equivalence principle and can talk about the dynamics quite confidently. We have,

$$a = \frac{\bar{a}}{\sqrt{\phi}} = \frac{A \ t^{2/(3+\alpha)}}{(1 - \phi_0 t)} \ . \tag{4.37}$$

Also, the Hubble parameter and the deceleration parameter q in the original version comes out as,

$$H = \frac{2}{3 + \alpha} \frac{1}{t} + \frac{\phi_0}{1 - \phi_0 t} \ , \tag{4.38}$$

$$q = -1 + \frac{\frac{2}{3+\alpha}(1 - \phi_0 t)^2 - \phi_0^2 t^2}{\left[\frac{2}{3+\alpha}(1 - \phi_0 t) + \phi_0 t \right]^2} \ . \tag{4.39}$$

From equations (4.37) and (4.38) it is evident that at $t \to \frac{1}{\phi_0}$, both a and H blow up together giving a Big Rip. However, this rip has a different characteristic than that engineered by a normal phantom field. In the latter, ρ_m goes to zero but ρ_{DE} goes to infinity at the rip. In the present case, however, there is no dark energy as such, and the scalar field is a part of geometry and hence it is difficult to recognize its contribution to the energy density. In the revised version, however, $\frac{2\omega+3}{4}\dot{\psi}^2$ is the contribution towards the stress tensor. It turns out

that at $t \to \frac{1}{\phi_0}$, this contribution remains quite finite. So the big rip is brought into being by the interaction, and not by a singularity in the stress tensor.
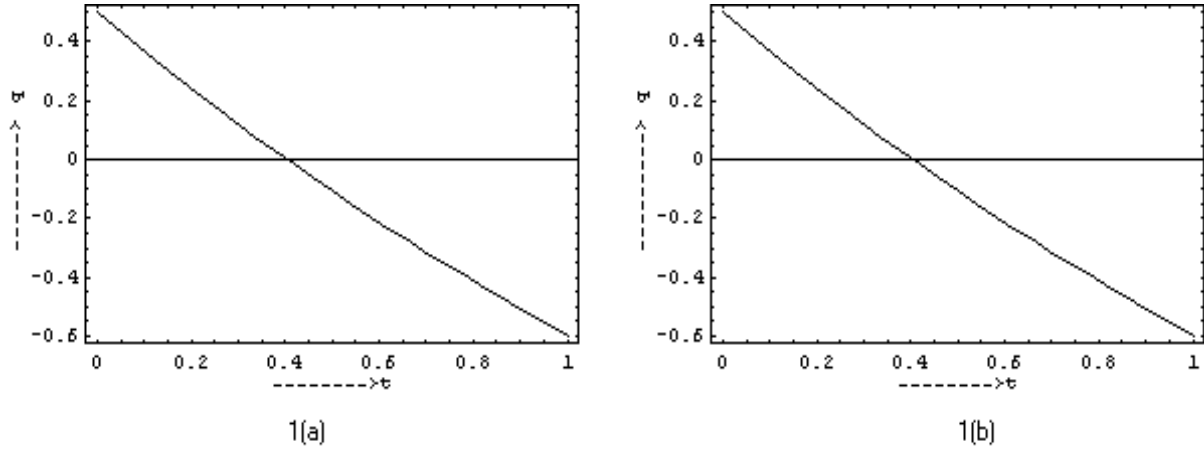

1(a)   1(b)

Figure 4.1: *Figure 4.1(a) and 4.1(b) shows the plot of q vs. t for different values of α. For figure 4.1(a) we choose α = 0.0000003 whereas for figure 4.2(b) we set the value as α = 0.0000001.*

The plot of q against t (figure 4.1) reveals that the deceleration parameter indeed has a sign flip in the desired direction and indicates an early deceleration ($q > 0$) followed by a late time acceleration ($q < 0$) of the universe. Also, the nature of the curve is not crucially sensitive to the value of α chosen.

Equation (4.27) clearly indicates that $\ddot{\bar{a}}/\bar{a}$ is negative definite. So the signature flip in $q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2}$ in the original Jordan frame must come from the time variation in ϕ via equation (4.37). Local astronomical experiments suggest that the present variation of G and hence that of ϕ has a very stringent upper bound. It is therefore imperative to check whether the present model is consistent with that bound. For figure 4.1, the value of the constant of integration ϕ_0 is fixed at 0.3. With this value, and the age of the universe taken as $\sim$ 15 Giga years, equation (4.35) yields

$$\left.\left|\frac{\dot{\phi}}{\phi}\right|\right|_0 \sim 10^{-10} \text{ per year,}$$

which is consistent with the requirements of the local experiments [31]. The suffix 0 indicates the present value. Also in this model, from equations (4.34) and (4.37), we get as $\omega \to \infty$, $\phi \to 1$ and $a \to t^{2/(3+\alpha)}$. Therefore, for very small value of α ($\sim 10^{-7}$), a is indistinguishable from that in GR ($a \sim t^{2/3}$). This is consistent with the notion that BD theory yields GR in the infinite ω limit.

4.2.3 Statefinder Parameters for the Model

Recently Sahni et al. [57, 58] have introduced a pair of new cosmological parameters {r, s}, termed as "statefinder parameters". These parameters can effectively differentiate between different forms of dark energy and provide a simple diagnostic regarding whether a particular model fits into the basic observational data. These parameters are

$$r = \frac{\dddot{a}}{aH^3} \text{ and } s = \frac{r-1}{3(q-\frac{1}{2})} \ .$$

Accordingly, we find the statefinder parameters for the present model as

$$r = 1 + \frac{3\phi_0^2 t^2 - 3\beta(1-\phi_0 t)^2}{[\beta + \phi_0 t(1-\beta)]^2} + \frac{2\beta(1-\phi_0 t)^3 + 2\phi_0^3 t^3}{[\beta + \phi_0 t(1-\beta)]^3} \tag{4.40}$$

and

$$s = \frac{3[\phi_0^2 t^2 - \beta(1-\phi_0 t)^2][\beta + (1-\beta)\phi_0 t] + 2\beta(1-\phi_0 t)^3 + 2\phi_0^3 t^3}{3[\beta + (1-\beta)\phi_0 t][-\frac{3}{2}\{\beta + (1-\beta)\phi_0 t\} + \beta(1-\phi_0 t)^2 - \phi_0^2 t^2]^2} \tag{4.41}$$

where $\beta = \frac{2}{3+\alpha}$.

If we now plot $r(s)$ for some small value of α ($\alpha << 1$), we find that the nature of the curve is similar to the one expected for scalar field quintessence models with equation of state parameter w in the range $-1 < w < 0$ [57, 58].
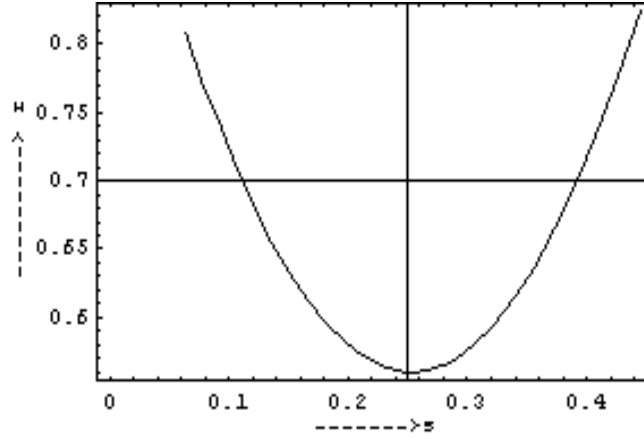
Figure 4.2: *Plot of r as a function of s for* $\alpha = .0000001$.

4.2.4   Discussion

Thus we see that for a spatially flat universe $(k = 0)$, we can construct a presently accelerating

model in Brans - Dicke theory or more precisely in a generalized version of it ($\omega = \omega(\phi)$) if

one considers an interaction between dark-matter and the geometrical scalar field. It deserves

mention that for this interaction, the Lagrangian should also be modified by the inclusion

of an interference term between ϕ and L_m. The salient feature of the model is that no dark

energy component is required. The nature of the q vs. t curve is not crucially sensitive to small

changes in the value of α, the parameter which determines the strength of the interaction;

only the time of 'onset' of acceleration would change by small amounts with α. In revised

unit, this interaction can be switched off by putting $\alpha = 0$. However, in Jordan frame it is not

possible to switch off the interaction with this particular choice. If we put $\alpha = 0$, the scalar

field itself becomes trivial. In this frame, the conservation equation (equation (4.28) along

with equation (4.30)) transforms to

$$\dot{\rho}_m + 3H\rho_m = -\left[\alpha H + \frac{\alpha - 1}{2}\frac{\dot{\phi}}{\phi}\right]\rho_m \ . \tag{4.42}$$

As one has both $H\rho_m$ and $\frac{\dot{\phi}}{\phi}\rho_m$ in the right hand side, the transfer of energy between matter and scalar field takes place both due to the expansion of the universe and the evolution of the scalar field. This equation shows that if

$$\phi = constant \ a^{-2\alpha/(\alpha-1)}$$

the interaction vanishes. In this case, the transfer of energy due to $H\rho_m$ and $\frac{\dot{\phi}}{\phi}\rho_m$ cancel each other.

It is evident from equation (4.33) that if we consider the interaction between dark matter and the geometrical field of the form considered in equation (4.30), a constant ω will give rise to an ever accelerating or ever decelerating model and definitely we are not interested in that. So, the idea of varying ω is crucial here as it can very well serve the purpose of providing a signature flip in q in this interacting scenario. It deserves mention that the specific choice for the interaction in equation (4.30) and the choice of $\omega = \omega(\phi)$ in equation (4.34) are taken so as to yield the desired result. This is indeed a toy model which simply shows that investigations regarding an interaction amongst matter and the nonminimally coupled scalar field is worthwhile.

From equation (4.33) it is also evident that $(2\omega + 3)$ has to be positive definite, i.e, ω has to pick up some positive value or at least ω should be greater than $-\frac{3}{2}$ in order to sustain a consistent model. Also, the parameter ω does not have any stringent limit and thus it may be possible to adjust the value of ω to some higher value. Equation (4.34) indicates that if ϕ is very close to unity, which is consistent with the present value of G, ω can attain a high value at the present epoch. This is encouraging as it might be possible to obtain a model which exhibits early deceleration and late time acceleration even with a high value of ω, compatible with the limit imposed on it by the solar system experiments [31].

Also, from equation (4.35) it is evident that $\frac{\dot{\phi}}{\phi} < 0$ and $\frac{\ddot{\phi}}{\phi} > 0$. So, from equation (4.24), which is of particular interest in studying the dynamics of the universe, we see that the last term $2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}$ is negative and is the key factor in driving the present acceleration of the universe. This term basically provides the effective negative pressure and becomes dominant during the later stages of evolution and drives an accelerated expansion. We have also calculated the statefinder parameters for the model and show that the {r, s} pair mimics that of a quintessence model. For this however, the constant α should be given a very small value ($\sim 10^{-7}$). In this model we have considered a particular form of interaction which indeed is not unique, and some complicated kind of interaction may lead to more viable solutions of the various cosmological problems, particularly to a model which is not restricted in future.

Although it is true that General Relativity (GR) is by far the best theory of gravity and the natural generalisation of BD theory to GR for large ω limit is shown to be restricted in some sense [30], still BD theory always seems to be ready to provide some useful clues to the solution of various cosmological problems. The solution to the "graceful exit" problem of inflation in terms of an 'extended inflation' scenario [24, 25] was first obtained in BD theory which provided hints towards the subsequent resolutions of the problem in GR. Once again here, BD theory in its own right could provide a model exhibiting the present cosmic acceleration without introducing any exotic dark energy component and since nothing definite is known about the source of this acceleration, this type of investigations may lead to some track along which viable solutions may finally be arrived at. However, this is a primitive model. This only shows that such investigations can be useful. It remains to be seen if the solution is an attractor, and whether the model is consistent with the structure formation.

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Chapter 5

Curvature Driven Acceleration

5.1 Curvature-driven Acceleration : a Utopia or a Reality?

(Journal reference : S. Das, N. Banerjee and N. Dadhich, Class. Quantum Grav.,23, 4159 (2006).)

5.1.1   Introduction

The search for a dark energy component, the driver of the present accelerated expansion of the universe, has gathered a huge momentum because the alleged acceleration is now believed to be a certainty, courtesy the WMAP data [1]. As no single candidate enjoys a pronounced supremacy over the others as the dark energy component in terms of its being able to explain all the observational details as well as having a sound field theoretic support, any likely candidate deserves a careful scrutiny until a final unambiguous solution for the problem emerges. The cosmological constant Λ, a minimally coupled scalar field with a potential, Chaplygin gas or even a nonminimally coupled scalar field are amongst the most popular candidates (see [2] for a comprehensive review). Recently an attempt in a slightly different direction is gaining more and more importance. This effort explores the possibility of whether geometry in its own right could serve the purpose of explaining the present accelerated expansion. The idea actually stems from the fact that higher order modifications of the Ricci curvature R, in the form of R^2 or $R_{\mu\nu}R^{\mu\nu}$ etc. in the Einstein - Hilbert action could generate an accelerated expansion in the very early universe [3]. As the curvature R is expected to fall off with the evolution, it is an obvious question if inverse powers of R in the action, which should become dominant during the later stages, could drive a late time acceleration.

A substantial amount of work in this direction is already there in the literature. Capozziello et al. [4] introduced an action where R is replaced by R^n and showed that it leads to an accelerated expansion, i.e, a negative value for the deceleration parameter q for $n = -1$ and $n = \frac{3}{2}$. Carroll et al. [5] used a combination of R and $\frac{1}{R}$, and a conformally transformed version of theory, where the effect of the nonlinear contribution of the curvature is formally taken care of by a scalar field, could indeed generate a negative value for the deceleration parameter.

Vollick also used this $1/R$ term in the action [6] and the resulting field equations allowed an asymptotically exponential and hence accelerated expansion. The dynamical behaviour of R^n gravity has been studied in detail by Carloni et.al [7]. A remarkable result obtained by Nojiri and Odinstov [8] shows that it may indeed be possible to attain an inflation at an early stage and also a late surge of accelerated expansion from the same set of field equations if the modified Lagrangian has the form $L = R + R^m + R^{-n}$ where m and n are positive integers. However, the solutions obtained are piecewise, i.e, large and small values of the scalar curvature R, corresponding to early and late time behaviour of the model respectively, are treated separately. But this clearly hints towards a possibility that different modes of expansion at various stages of evolution could be accounted for by a curvature driven dynamics. Other interesting investigations such as that with an inverse $\sinh(R)$ [9] or with $\ln R$ terms [10] in the action are also there in the literature.

The question of stability [11] and other problems notwithstanding, these investigations surely open up an interesting possibility for the search of dark energy in the non-linear contributions of the scalar curvature in the field equations. However, in most of these investigations so far mentioned, the present acceleration comes either as an asymptotic solution of the field equations in the large cosmic time limit, or even as a permanent feature of the dynamics of the universe. But both the theoretical demand [12] as well as observations [13] (see also [1]) clearly indicate that the universe entered into its accelerated phase of expansion only very recently and had been decelerating for the major part of its evolution. So the deceleration parameter q must have a signature flip from a positive to a negative value only in a recent past.

In the present work, we write down the field equations for a general Lagrangian $f(R)$ and

investigate the behaviour of the model for two specific choices of $f(R)$, namely $f(R) = R - \frac{\mu^4}{R}$ and $f(R) = e^{-\frac{R}{6}}$.

Although the field equations, a set of fourth order differential equations for the scale factor a, could not be completely solved analytically, the evolution of the 'acceleration' of the universe could indeed be studied at one go, i.e, without having to resort to a piecewise solution. The results obtained are encouraging, both the examples show smooth transitions from the decelerated to the accelerated phase. In this work we virtually assume nothing regarding the relative strengths of different terms and let them compete in their own way, and still obtain the desired transition in the signature of the deceleration parameter q. This definitely provides a very strong support for the host of investigations on curvature driven acceleration, particularly those quoted in [5, 6, 7 and 8].

In the evolution equation, q is expressed as a function of H, the Hubble parameter. This enables one to write an equation with only q to solve for; as the only other variable remains is H which becomes the argument. This method appears to be extremely useful, although it finds hardly any application in the literature. The only example noted by us is the one by Carroll et al. [14], which, however, describes the nature only in an asymptotic limit.

In the next section the model with two examples are described and in the last section we include some discussion.

5.1.2 Curvature Driven Acceleration

The relevant action is

$$A = \int \left[\frac{1}{16\pi G} f(R) + L_m \right] \sqrt{-g} \, d^4 x, \tag{5.1}$$

where the usual Einstein - Hilbert action is generalized by replacing R with $f(R)$, which is an analytic function of R, and L_m is the Lagrangian for all the matter fields. A variation of this action with respect to the metric yields the field equations as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}{}^c + T_{\mu\nu}{}^M, \tag{5.2}$$

where the choice of units $8\pi G = 1$ has been made. $T_{\mu\nu}{}^M$ represents the contribution from matter fields scaled by a factor of $\frac{1}{f'(R)}$ and $T_{\mu\nu}{}^c$ denotes that from the curvature to the effective stress energy tensor. $T_{\mu\nu}{}^c$ is actually given as

$$T_{\mu\nu}{}^c = \frac{1}{f'(R)}\left[\frac{1}{2}g_{\mu\nu}(f(R) - Rf'(R)) + f'(R)^{;\alpha\beta}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\nu}g_{\alpha\beta})\right]. \tag{5.3}$$

The prime indicates differentiation with respect to Ricci scalar R. It deserves mention that we use a variation of (5.1) w.r.t. the metric tensor as in Einstein - Hilbert variational principle and not a Palatini variation where A is varied w.r.t. both the metric and the affine connections. As the actual focus of the work is to scrutinize the role of geometry alone in driving an acceleration in the later stages, we shall work without any matter content, i.e, $L_m = 0$ leading to $T_{\mu\nu}{}^M = 0$. So for a spatially flat Robertson - Walker spacetime, where

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2], \tag{5.4}$$

the field equations (5.2) take the form (see [4])

$$3\frac{\dot{a}^2}{a^2} = \frac{1}{f'}\left[\frac{1}{2}(f - Rf') - 3\frac{\dot{a}}{a}\dot{R}f''\right], \tag{5.5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{f'}[2\frac{\dot{a}}{a}\dot{R}f'' + \ddot{R}f'' + \dot{R}^2f''' - \frac{1}{2}(f - Rf')]. \tag{5.6}$$

Here a is the scale factor and an overhead dot indicates differentiation w.r.t. the cosmic time t. If $f(R) = R$, the equation (5.2) and hence (5.5) and (5.6) take the usual form of vacuum

Einstein field equations. It should be noted that the Ricci scalar R is given by

$$R = -6 \left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right] ,\tag{5.7}$$

and already involves a second order time derivative of a. As equation (5.6) contains $\ddot{R}$, one actually has a system of fourth order differential equations.

It deserves mention at this stage that if R is a constant, then whatever form of $f(R)$ is chosen except $f(R) = R$, equations (5.5) and (5.6) represent a vacuum universe with a cosmological constant and hence yield a deSitter solution, i.e, an ever accelerating universe. Evidently we are not interseted in that, we are rather in search of a model which clearly shows a transition from a decelerated to an accelerated phase of expansion of the universe. As we are looking for a curvature driven acceleration at late time, and the curvature is expected to fall off with the evolution, we shall take a form of $f(R)$ which has a sector growing with the fall of R. We work out two examples where indeed the primary purpose is served.

(i) $f(R) = R - \frac{\mu^4}{R}$.

In the first example, we take

$$f(R) = R - \frac{\mu^4}{R} ,\tag{5.8}$$

where μ is a constant. Indeed μ has a dimension, that of $R^{\frac{1}{2}}$, i.e, that of $(time)^{-1}$. This is exactly the form used by Carroll et al. [5] and Vollick [6]. Using the expression (5.8) in a combination of the field equations (5.5) and (5.6), one can easily arrive at the equation

$$2\dot{H} = \frac{1}{(R^2 + \mu^4)} \left[2\mu^4 \frac{\ddot{R}}{R} - 6\mu^4 \frac{\dot{R}^2}{R^2} - 2\mu^4 H \frac{\dot{R}}{R} \right] ,\tag{5.9}$$

where $H = \frac{\dot{a}}{a}$, is the Hubble parameter. As both R and H are functions of a and its derivatives, equation (5.9) looks set for yielding the solution for the scale factor. But it involves fourth order derivatives of a (R already contains $\ddot{a}$) and is highly nonlinear. This makes it difficult to obtain a completely analytic solution for a. As opposed to the earlier investigations where either a piecewise or an asymptotic solution was studied, we adopt the following strategy. The point of interest is the evolution of the deceleration parameter

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1. \tag{5.10}$$

So we translate equation (5.9) into the evolution equation for q using equation (5.10) and obtain

$$\mu^4 \frac{\ddot{q}}{(q-1)} - 3\mu^4 \frac{\dot{q}^2}{(q-1)^2} - 6\mu^4 H^2 (q+1)^2 + 3\mu^4 H^2 (q+1)$$

$$+ \ 3\mu^4 H \frac{(2q+3)}{(q-1)}\dot{q} + 36 H^6 (q-1)^2 (q+1) = 0. \tag{5.11}$$

This equation, although still highly nonlinear, is a second order equation in q. But the problem is that both q and H are functions of time and cannot be solved for with the help of a single equation. However, they are not independent and are connected by equation (5.10). So we replace time derivatives by derivatives w.r.t. H using equation (5.10) and write (5.11) as

$$\frac{1}{3}(q^2-1)H^2 q^{\dagger\dagger} - \frac{2}{3}H^2(q+2)q^{\dagger 2} - \frac{1}{3}H(q-1)(4q+7)q^{\dagger} - (2q+1)(q-1)^2 + H^4(q-1)^4 = 0. \tag{5.12}$$

Here for the sake of simplicity μ^4 is chosen to be 12 (in proper units), and a dagger represents a differentiation w.r.t. the Hubble parameter H. As $\frac{1}{H}$ is a measure of the age of the universe and H is a monotonically decreasing function of the cosmic time, equation (5.12) can now be used as the evolution equation for q. The equation appears to be hopelessly nonlinear to give an analytic solution but if one provides two initial conditions, for q and $q^{\dagger}$, for some value of

H, a numerical solution is definitely on cards. We choose units so that H_0, the present value of H, is unity and pick up sets of values for q and $q^\dagger$ for $H = 1$ (i.e, the present values) from observationally consistent region [15] and plot q versus H numerically in figure 5.1. As the inverse of H is the estimate for the cosmic age, 'future' is given by $H < 1$ and past by $H > 1$. The plots speak for themselves. One has the desired feature of a negative q at $H = 1$ and it comes to this negative phase only in the recent past. An important point to note here is that neither the nature of the plots, nor the values of H at which the transition takes place, crucially depends on the choice of initial conditions, so the model is reasonably stable.
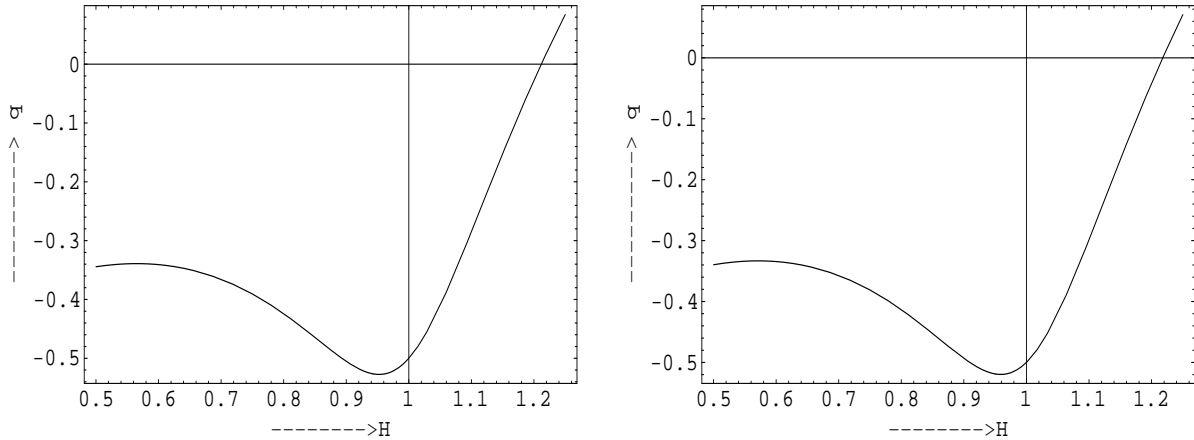

Figure 5.1: *Figure 5.1(a) and 5.1(b) shows the plot of q vs. H for $f(R) = R - \frac{\mu^4}{R}$ for different initial conditions. For figure 5.1(a) we choose the initial conditions as $q[1] = -0.5$, $q'[1] = 1.2$ whereas for figure 5.1(b) we set the initial conditions as $q[1] = -0.5$, $q'[1] = 1.0$.*

(ii) $f(R) = e^{-\frac{R}{6}}$.

In this choice, the function f is monotonically increasing with t as R is decreasing with t.

The field equations (5.5) and (5.6) have the form

$$3\frac{\dot{a}^2}{a^2} = -6\left[\frac{1}{2}(1+\frac{R}{6}) - \frac{1}{12}\frac{\dot{a}}{a}\dot{R}\right] ,$$

(5.13)

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -6\left[\frac{1}{2}(1+\frac{R}{6}) - \frac{1}{18}\frac{\dot{a}}{a}\dot{R} - \frac{1}{36}\ddot{R} + \frac{1}{216}\dot{R}^2\right].$$

(5.14)

From these two equations it is easy to write

$$2\dot{H} = \frac{1}{6}\ddot{R} - \frac{1}{36}\dot{R}^2 - \frac{1}{6}H\dot{R}.$$

(5.15)

Following the same method as before, the evolution of q as a function of H can be written as

$$H^4(q+1)q^{\dagger\dagger} + [H^4 - (q+1)H^6]q^{\dagger 2} + [2(q+1)H^3 + 6qH^3 + 3H^3 - 4H^5(q^2-1)]q^{\dagger}$$

$$+ 6H^2q^2 + 2H^2q - 4H^4(q^2-1)(q-1) - 8H^2 + 2 = 0 .$$

(5.16)

With similar initial conditions for q and $q^{\dagger}$ at $H = 1$, the plot of q versus H (figure 5.2) shows features similar to the previous example, the deceleration parameter q has a signature change, from a positive to a negative phase in the recent past ($H > 1$). This example has an additional feature that the universe re-enters a decelerated phase of expansion again in a near future ($H < 1$). In this case also, a small change in initial conditions hardly has any perceptible change in the graphs.

In this case, a curvature singularity in a finite future is indicated. As $q = -\frac{\ddot{a}/a}{\dot{a}^2/a^2}$ and H remains finite, $\frac{\ddot{a}}{a}$ and hence the curvature (via equation (5.7)) has a singularity in a finite future. This is consistent with [5] which indicates that a curvature quintessence may end up with three possibilities - an asymptotic de Sitter, a power law inflation or a curvature singularity in a finite future. The present case corresponds to the third possibility. The curves however show quite clearly that this singularity is not a 'Big Rip' type, where due to

continuous vigorous acceleration both H and a blow up in some finite future. Here the model clearly enters into a decelerating phase close to $H = 0.8$, as shown by the figure.

From equation (5.16), one can also conclude that for very high value of H (i.e, when the age of the universe was very small), $q \rightarrow -1$, which gives an early inflation.
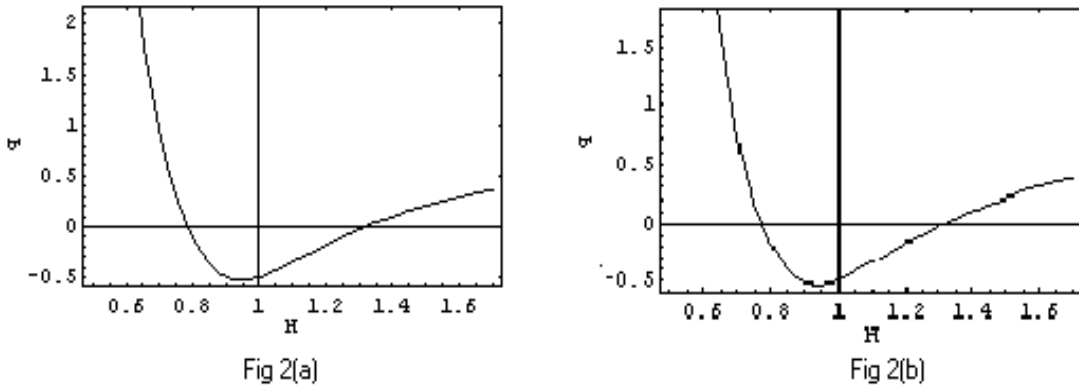

Fig 2(a)   Fig 2(b)

Figure 5.2: *Figure 5.2(a) and 5.2(b) shows the plot of q vs. H for $f(R) = e^{-\frac{R}{6}}$ for different initial conditions. Here also for figure 5.2(a) and 5.2(b) we set the initial conditions as $q[1] = -0.5$, $q'[1] = 1.0$ and $q[1] = -0.5$, $q'[1] = 1.2$ respectively.*

As the plots provide a sufficient data set, attempts could be made to find the closest analytical expression for $q = q(H)$. These expressions are found to be polynomials. For example, a very close analytical expression for figure 5.2(b), within the accuracy of plots, is given as

$$q = 47.95H^6 - 335.73H^5 + 991H^4 - 1586.90H^3 + 1459.20H^2 - 729.73H + 153.74. \quad (5.17)$$

This expression holds only when H is reasonably close to one, and has nothing to do with other ranges of values of H.

5.1.3 Discussion

The present work indicates that by asking the question whether geometry in its own right can lead to the late surge of accelerated expansion, some feats can surely be achieved. Both the examples considered here indicate that one can build up models which start accelerating at the later stage of evolution and thus allow all the past glories of the decelerated model like nucleosynthesis or structure formation to remain intact. An added bonus of the second example is that the universe re-enters a decelerated phase in near future and the 'phantom menace' is avoided - the universe does not have to have a singularity of infinite volume and infinite rate of expansion in a 'finite' future.

It is of course true that a lot of other criteria have to be satisfied before one makes a final choice, and we are nowhere near that. Already there is a criticism of $\frac{1}{R}$ gravity that it is unsuitable for local astrophysics because of problems regarding stability [11]. However, it was pointed out by Nojiri and Odinstov [8] that a polynomial may save the situation (see also reference [16]). Our second example is exponential in R, i.e, a series of positive powers in R, and hence could well satisfy the criterion of stability. As already pointed out, although the choice of $f(R) = R - \frac{\mu^4}{R}$ is already there in the literature and served the purpose in a restricted sense than it does in the present work, the choice of $f(R) = e^{-\frac{R}{6}}$ has hardly any mention in the literature. The Lagrangian $f(R) = e^{-\frac{R}{6}}$ contains a cosmological constant as $f(R) \approx 1 - \frac{R}{6}$ for small R. So indeed one expects that it gives an accelerated expansion. But the interesting feature is that the same model gives an early inflation followed by a decelerated expansion, then an accelerated expansion around the present epoch and a decelerated phase once again in near future.

It should also be noted that the present toy model deals with a vacuum universe and one

has to either put in matter, or derive the relevant matter at the right epoch from the curvature itself. Some efforts towards this have already begun [17]. On the whole, there are reasons to be optimistic about a curvature-driven acceleration which might become more and more important in view of the fact that WMAP data could indicate a very strong constraint on the variation of the equation of state parameter w [18].

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